# PROBLEM SET 8, 16300 SECTION 21

Due Friday May 29 in class.

1. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 5 & 6 \\ 3 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \\ 2 & 5 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$ and $w = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Which of these matrices and vectors are we allowed to multiply? (For example, we can multiply $Cw$, but not $CB$ or $wC$.) In each case where we are allowed to multiply, calculate the product. You should get 7 answers.

2. Exercise 2.3.3 from Sally.

3. Exercise 2.3.6 from Sally.

4. Given an $m \times n$ matrix $A = (a_{ij})$, we define the *transpose* ${}^tA$ of $A$ as the $n \times m$ matrix with $ij$'th entry $a_{ji}$. Show that ${}^t(AB) = {}^tB{}^tA$.

Bonus: Define $V^* = \mathcal{L}(V, F)$. Then $V^*$ is a vector space of the same dimension as $V$, and given a basis $\{v_1, \ldots, v_n\}$ for $V$ we get a corresponding basis $\{v_1^*, \ldots, v_n^*\}$ for $V^*$ where $v_i^*(\alpha_1 v_1 + \ldots + \alpha_n v_n) = \alpha_i$. Given a linear transformation $T : V \to W$, show that there is an induced linear transformation $T^* : W^* \to V^*$ sending $f : W \to F$ to the composite $f \circ T : V \to F$. Show that if $A$ is the matrix for $T$ then ${}^tA$ is the matrix for $T^*$, using the corresponding basis for $V^*$ and $W^*$.

5. Exercise 2.4.10 from Sally.

6. Exercise 2.4.13 from Sally.

7. Problem 28-7 from Spivak. What does this have to do with determinants and inverse matrices?