

## PROBLEMSET 8, 16300 SECTION 21

Due Tuesday May 27 at 5pm in the mailbox marked “Angeltveit  
16100-16200-16300 section 21” in the basement.

1. a) Let  $F$  be a field and let  $V$  be a vector space over  $F$ . Let  $0$  be the additive identity for  $F$  and let  $\mathbf{0}$  be the additive identity for  $V$ . Prove that  $0 \cdot v = \mathbf{0}$  for any  $v \in V$ .

b) Similarly, prove that  $(-1) \cdot v = -v$ .

c) Observe that from a) and b) we get  $(1 + (-1)) \cdot v = 1 \cdot v - v = \mathbf{0}$ , so  $1 \cdot v = v$ . Does that mean that property e on p. 51, which says  $1 \cdot v = v$ , follows from the others (A1-A5 and a-d)? If so, explain. If not, can you come up with an example of a field  $F$ , a set  $V$  and an action  $\cdot : F \times V \rightarrow V$  which satisfies A1-A5 and a-d but not e?

2. Exercise 2.1.14 from Sally’s notes.

3. Exercise 2.1.18 from Sally’s notes. If you don’t already know that  $\mathbb{Q}$  is a countable set, ask me about it.

4. Exercise 2.1.30 from Sally’s notes. For part iii, the set  $\{f_x\}$  for  $x \in X$ , where  $f_x(y) = 0$  if  $y \neq x$  and  $1$  if  $y = x$  is not quite a basis. For example, you cannot write the function  $f(x) = 1$  for all  $x$  as a finite linear combination of these when  $X$  is infinite. Don’t spend too much time on this.

5. Exercise 2.2.2 from Sally’s notes.

6. Exercise 2.2.3 from Sally’s notes.

7. Exercise 2.2.12 from Sally’s notes

8. Suppose  $V$  is a vector space over  $F$  of dimension  $m$  and  $W$  is a vector space over  $F$  of dimension  $n$ . What is the dimension of the vector space  $\mathcal{L}(V, W)$  of linear transformations from  $V$  to  $W$ ? Given a basis for  $V$  and  $W$ , can you write down a basis for  $\mathcal{L}(V, W)$ ?