Geom/Top: Homework 5 (due Monday, 11/19/12)

1. Read Farb notes.
2. Read along in Hatcher.

1. Hatcher, Section 3.2, Problems 3, 12, 15 (compute the Poincaré series of $S^n$, $\mathbb{RP}^n$, $\mathbb{RP}^\infty$ only).

2. Compute the cohomology ring of $\mathbb{RP}^2 \times \mathbb{RP}^2$.

3. Prove that the Klein bottle and the space $X = \mathbb{RP}^2 \vee S^1$ have isomorphic cohomology rings with $\mathbb{Z}$ coefficients, but not with $\mathbb{Z}/2\mathbb{Z}$ coefficients.

4. Let $L_1 \subset S^3$ be the union of two disjoint round circles that link each other once. Let $L_2$ be the union of two disjoint, unlinked round circles. Prove that $S^3 - L_1$ and $S^3 - L_2$ has isomorphic integral cohomology groups, but not isomorphic cohomology rings.