Geom/Top: Homework 5 (due Monday, 11/12/12)

1. Read Farb notes.
2. Read along in Hatcher.

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1. Let $\{G_n\}$ be any sequence of finitely generated abelian groups. Prove that there is a CW complex X, with finite n-skeleton for each n, so that $H_n(X) = G_n$ for $n \geq 1$.

2. Find spaces $X = A \cup B$ and $X' = A' \cup B'$ so that for all $n \geq 0$:

   $$H_n(A) \approx H_n(A') \quad \text{and} \quad H_n(B) \approx H_n(B') \quad \text{and} \quad H_n(A \cap B) \approx H_n(A' \cap B')$$

   but with some $j > 0$ for which $H_j(X)$ not isomorphic to $H_j(X')$. Thus the "connecting homomorphism" in the Mayer-Vietoris sequence does matter when computing the homology of X.

3. Solve the bonus problem on the exam.

4. (a) Compute the homology groups of $\mathbf{R}^2$ minus k-points, for any $k \geq 1$.

   (b) Compute the homology of $\mathbf{R}^3$ minus k non-intersecting lines.

5. Let $U(2)$ denote the group of complex 2×2 unitary matrices. Endow $U(2)$ with the subspace topology $U(2) \subset \mathbf{C}^4$.

   (a) Let $SU(2)$ be the subgroup (and topological subspace) of $U(2)$ consisting of those matrices of determinant 1. Prove that $SU(2)$ is homeomorphic to S^3.

   (b) Consider the determinant $\det : U(2) \to S^1$ map. Let $A \subset U(2)$ (resp. B) be the subset of elements with determinant $e^{i\theta}, -\pi < \theta < \pi$ (resp. $0 < \theta < 2\pi$). Compute $H_n(U(2))$ for all $n \geq 2$ by applying Mayer-Vietoris to $X = A \cup B$.

6. Hatcher, §3.1, Problems 4,5,

Extra Credit Problems

1. Let $\mathrm{Conf_n}(\mathbf{C})$ be the set of (unordered) configurations of n distinct points in $\mathbf{C}$, topologized as a subset of $\mathbf{C}^n$.

   (a) Prove that $\mathrm{Conf_n}(\mathbf{C})$ is the space of monic, square-free, polynomials of degree n. [Worth 2 points.]

   (b) Prove that $\mathrm{Conf_n}(\mathbf{C})$ is the complement in $\mathbf{C}^n$ of a collection of hyperplanes. [Worth 2 points.]

   (c) Compute the homology of $\mathrm{Conf_n}(\mathbf{C})$. [Worth 30 points.]