

$$\begin{array}{ccccc}
f_! Q \Delta_A^* R(f^* X \bar{\wedge} Y) & \longrightarrow & f_! Q \Delta_A^* R(f \times f)^*(X \bar{\wedge} f_! Y) & \longrightarrow & f_! Q \Delta_A^* R(f \times f)^* R(X \bar{\wedge} f_! Y) \\
\uparrow \sim & & \uparrow & & \uparrow \sim \\
f_! Q \Delta_A^*(f^* X \bar{\wedge} Y) & \longrightarrow & f_! Q \Delta_A^*(f \times f)^*(X \bar{\wedge} f_! Y) & \longrightarrow & f_! Q \Delta_A^*(f \times f)^* R(X \bar{\wedge} f_! Y) \\
\downarrow \sim & & \downarrow & & \downarrow \cong \\
f_! \Delta_A^*(f^* X \bar{\wedge} Y) & \longrightarrow & f_! \Delta_A^*(f \times f)^*(X \bar{\wedge} f_! Y) & \longrightarrow & f_! \Delta_A^*(f \times f)^* R(X \bar{\wedge} f_! Y) & \longleftarrow & f_! Q f^* \Delta_B^* R(X \bar{\wedge} f_! Y) \\
\downarrow \cong & & \downarrow \cong & & \downarrow \cong & \searrow \cong & \downarrow \cong \\
\Delta_B^*(X \bar{\wedge} f_! Y) & \xrightarrow{\text{?}} & \Delta_B^*(X \bar{\wedge} f_! Y) & \longrightarrow & \Delta_B^*(X \bar{\wedge} f_! Y) & \xrightarrow{\cong} & \Delta_B^*(X \bar{\wedge} Q f_! Y) \\
& & \text{(*)} & & & & \downarrow \\
& & f_! f^* \Delta_B^*(X \bar{\wedge} f_! Y) & \longrightarrow & \Delta_B^*(X \bar{\wedge} Q f_! Y) & & \Delta_B^*(X \bar{\wedge} Q f_! Y) \\
& & & & & & \downarrow \\
& & & & & & \Delta_B^*(X \bar{\wedge} Q R f_! Y) \\
& & & & & & \downarrow \\
& & & & & & \Delta_B^*(X \bar{\wedge} R f_! Y)
\end{array}$$

We deal with the arrow on the bottom by replacing $f_! Y$ with $QRf_! Y$ as follows:

$$\begin{array}{ccc}
\Delta_B^*(X \bar{\wedge} f_! Y) & \xrightarrow{\text{?}} & \Delta_B^*(X \bar{\wedge} f_! Y) \\
\uparrow \sim & & \uparrow \sim \\
\Delta_B^*(X \bar{\wedge} Q f_! Y) & \longrightarrow & \Delta_B^*(X \bar{\wedge} Q f_! Y) \\
\downarrow \sim & & \downarrow \sim \\
\Delta_B^*(X \bar{\wedge} QR f_! Y) & \xrightarrow{\sim} & \Delta_B^*(X \bar{\wedge} QR f_! Y)
\end{array}$$