Sergei Adian (1/1/31-4/5/20)

Sergei Ivanovich Adian

Born: 1 January 1931 in Kushchi, Dashkesan District, Azerbaijan Soviet Republic

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to see eight larger pictures

Show birthplace location

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Sergei Ivanovich Adian was born in Kushchi, a mountain village forty kilometres from the city of Kirovabad (now Gjanja). His father, Ivan Arakelovich Adiyan, was born the son of a shepherd in 1908. Not having the opportunity to finish secondary school, Ivan became a carpenter and worked on local building sites. In 1930 he married Lusik, the 17-year-old daughter of a local farmer, Konstantin Truziyan. Two years later Sergei's parents moved to Kirovabad, where Ivan worked as a carpenter. Initially they rented a room, and it was only at the end of the 1930s that the father bought a plot of land in the centre of the city and built a small house with one room, a porch, and a little cellar. Being a builder, Ivan planned to add a second floor to the house, but this plan was disrupted by the war. By that time there were already four children in the family. The mother did not work, but the parents completed their secondary education studies at an Armenian evening school for labourers. Although at the time Sergei, like his parents, did not speak Russian, he was sent in 1938 to study at the Russian secondary school no. 11 in Kirovabad. His father insisted on that, since he believed that after finishing the school it would be easier for his son to get a higher education. And so from his very first year in school young Sergei had to develop persistence and diligence. His problems with Russian were overcome by the end of this first year.

In 1941, at the very beginning of the war, Sergei's father was conscripted. After a short training course somewhere in the northern Caucasus, he was sent to the front. There he did not survive for long and his family received notice that he was missing. Sergei's mother got a job selling mineral water in a kiosk, and 10-year-old Sergei effectively became the head of the household, helping his mother keep house and bring up his two younger brothers Semik (8 years old) and Yurik (3), and his sister Svetlana (6). There were good and, more importantly, exacting teachers in the school Sergei attended. His mathematical talent soon became apparent. Once, in the fourth grade, the teacher asked every student to solve one problem from a problem book, and she walked between the rows monitoring progress. While everybody else was still working hard on the first problem, Sergei had already solved several of them. The teacher was pleased and went on with the experiment until the end of the lesson. As a result, Sergei solved 40 problems in one lesson. Another interesting episode took place in the tenth and last grade. Before the spring break, as part of preparation for final examinations, another teacher of mathematics, the school headmaster, gave his students homework in solid geometry based on trigonometric formulae from the popular problem book by Rybkin. The teacher asked everyone to solve only a couple of problems from each section, and he was immensely surprised when one of the students, Sergei Adian, handed him a thick notebook with complete solutions, drawings included, of all the problems from Rybkin's book! It is not surprising that the Education Department of Kirovabad submitted to Baku, the capital of the Azerbaijan Republic, a petition to send Sergei Adian to Moscow State University (MSU) to continue his education after completing his secondary school studies. But in Baku his name was crossed off the list. Following his headmaster's recommendation, Sergei then went to Erevan intending to enter Erevan University. But 'national politics' prevented him registering because prospective students were supposed to pass a written examination in Armenian, and the Armenian alphabet was certainly not a subject taught in a Russian school located in Azerbaijan. Finally, in 1948 Adian had to enter the Russian Pedagogical Institute of Erevan. His education there lasted only one year. After that he and others among the best students in Erevan were sent to continue their studies in Moscow. This action was organised by the USSR Ministry of Education, and every student was transferred to an institution similar to the one he was taken from. As a consequence, Adian was refused admission to MSU, so again for technical reasons, another attempt to enter there failed. However, he later said:-

> *I should admit that at that time I was extremely lucky: I was not able to go to MSU. As fate willed, I went to the Moscow State Pedagogical Institute (MSPI), where I met Petr Sergeevich Novikov, and he introduced me to his wife Lyudmila Vsevolodovna ...*

If ever there was an encounter that could be called fortunate, it was the meeting of Adian and his future teacher, mentor, and friend (in spite of the difference in age) Petr Sergeevich Novikov. Adian started his research work at MSPI, with Novikov as his advisor, in the field of the descriptive theory of functions. In his first work as a student in 1950, he proved that the graph of a function $f(x)$ of a real variable satisfying the functional equation $f(x + y) = f(x) + f(y)$ and having discontinuities is dense in the plane. (Clearly, all continuous solutions of the equation are linear functions.) This result was not published at the time. It is curious that about 25 years later the American mathematician Edwin Hewitt from Seattle gave preprints of some of his papers to Adian during a visit to MSU, one of which was devoted to exactly the same result, which was published by Hewitt much later.

In his graduate work in 1953 relating to the theory of discontinuous functions, Adian constructed examples of semicontinuous functions on the interval [0, 1] that, for any partition of the interval into a countable number of subsets E_i, have discontinuities on at least one of the subsets upon restriction to this subset. This contribution also was not published right away. In 1958, following a proposal of Adian, the work was published in the Scientific Notes of MSPI as joint work with Novikov.

In the autumn of 1954, Novikov suggested to Adian (then in his third year of graduate study) that he work on the word problem for finitely presented groups, noting that though Adian's results already obtained in the theory of functions were certainly enough for a Ph.D. thesis, this new problem was more interesting, was mentioned in Kurosh's monograph, and was a difficult problem that had resisted solution by Novikov's methods. In suggesting it, Novikov considered the fact that Adian had already mastered thoroughly the methods of Novikov's proof, not yet published, of the unsolvability of the word problem. By the beginning of 1955 Adian had managed to prove the undecidability of practically all non-trivial invariant group properties, including the undecidability of being isomorphic to a fixed group G, for any group G. These results made up his Ph.D. thesis, defended in 1955 and first published the same year (the full text of the proof was published two years later). This is one of the most remarkable, beautiful, and general results in algorithmic group theory and is now known as the Adian-Rabin theorem (Michael O Rabin published a simpler proof of the result some years later).

Of course, the history of mathematics offers quite a few other examples of work of undergraduate or graduate students which later became classical results in their fields. However, what distinguishes the first published work by Adian even in this brilliant company is its completeness. In spite of numerous attempts, nobody has added anything fundamentally new to the results during the past 50 years. For this work Adian was awarded the Moscow Mathematical Society Prize in 1956 and the Chebyshev Prize of the Soviet

Academy of Sciences in 1963. It should be noted that A S Esenin-Volpin, one of the official opponents of Adian's Ph.D. thesis, after reading and verifying the thesis, made a special trip to Novikov's dacha in the summer of 1955 to convince him that such work merited a D.Sc. degree. Novikov answered that there was nothing to be concerned about: he did not doubt that Adian would write another work for his D.Sc. dissertation. And the first opponent, Anatoly Ivanovich Malcev, proposed that the Academic Council of MSPI, where the defence was taking place, should call special attention to the outstanding level of the work.

After completing his graduate studies, Adian worked for several years (in close cooperation with Novikov) as an assistant professor in the Mathematical Analysis Department of MSPI. And in 1957 an event happened which completely changed life for both him and his teacher, the Department of Mathematical Logic was created in the Steklov Mathematical Institute (MIAN), and Novikov was invited to lead it. Adian became one of the first members of this new department, and his subsequent research career was closely connected with it. Furthermore, the collaboration between Novikov and Adian on the Burnside problem started (about 1960) already within the precincts of MIAN. In 1960, at the insistence of Novikov and his wife Lyudmila Keldysh, Adian settled down to work on the Burnside problem. Completing the project took intensive efforts from both collaborators in the course of eight years, and in 1968 their famous paper *Infinite periodic groups* appeared, containing a negative solution of the problem for all odd periods $n > 4381$, and hence for all multiples of those odd integers as well. Adian published the classic monograph *The Burnside problem and identities in groups* (Russian) in 1975 (an English translation was published four years later).

In 1965, at the invitation of A A Markov, Adian also took a second position, in the Department of Mathematical Logic at MSU. His work there continues to ensure a close and fruitful collaboration of the department with the Department of Mathematical Logic at MIAN. In 1973, because of a serious illness of Novikov and at Novikov's personal request supported by Vinogradov, the director of MIAN, Adian was appointed head of the department. This appointment happened despite the fact that neither Adian nor Novikov were members of the Communist Party. The Department of Mathematical Logic in the Faculty of Mechanics and Mathematics at MSU went through a similar period of turbulence, for similar reasons, when the head of the department, A A Markov, fell sick at the end of the 1970s. In many respects due to the energy, integrity, and diplomatic skills of Adian, this situation was also resolved favourably for the department.

Adian has always devoted much attention to strengthening the Department of Mathematical Logic at MIAN, to training researchers in the Department of Mathematical Logic at MSU, and to developing new connections between these two related groups. He has had great success in this direction. Under his guidance more than thirty Ph.D. and D.Sc. dissertations have been written. His students are prominent researchers in algebra, mathematical logic, and computational complexity theory. After finishing at MSU, the strongest of them transferred to positions in the Department of Mathematical Logic at MIAN, which under his leadership became one of the most prominent and respected research centres in logic.

In the Department of Mathematical Logic at MSU Adian has for many years led a seminar on algorithmic problems of algebra and logic, in addition to sharing leadership of the department's main seminar with V A Uspenskii. Several times he has also given mandatory lecture courses in mathematical logic for the first and fourth years, and special lecture courses on algorithmic problems of algebra and on infinite periodic groups. Adian is in essence the creator and leader of a whole research school in mathematical logic and algorithmic problems of algebra. Besides his productive research and teaching activities, Adian is active in editorial and organizational work. As long ago as the end of 1950s, S M Nikol'skii invited Adian, at the suggestion of Novikov, to edit the section on mathematical logic in *Referativnyi Zhurnal: Matematika*, the Russian mathematical review journal, because there was then a huge backlog of articles to be reviewed. In the shortest possible time Adian rectified the situation there with respect to logic by mobilizing almost all his colleagues for the thankless task of writing reviews (for only a paltry fee). At about the same time, he drew attention to the fact that a remarkable textbook on mathematical logic written by Novikov had not been published, and that undergraduate and graduate students had to read a typescript. Novikov explained that the publishing house *Fizmatgiz* had rejected the manuscript because they had not liked the frequent use of the term Hilbert formalism in the preface: this was regarded as propaganda for a harmful bourgeois philosophical theory. Novikov planned to return the advance of his fee to the publishers. Adian told him that this was simply unacceptable and began to help in revising and editing the book. He declined Novikov's offer that he should be coauthor, and about half a year later the first edition of Novikov's textbook on mathematical logic appeared. The book was subsequently translated into several foreign languages.

For many years Adian was the head of the Specialized Scientific Council of Vysshaya Attestatsionnaya Komissiya (VAK, the Higher Certification Commission) concerned with defence of D.Sc. dissertations in mathematical logic, algebra, number theory, geometry, and topology, first as the vice-chairman and later, after the death of Vinogradov, as the chairman. In 1991 he asked to be relieved of the chairman position in view of his 60th birthday. However, when the directorate of MIAN proposed for this post a person who manifestly was not suitable, Adian could not reconcile himself with this and declared that he was prepared to remain in the position until a more appropriate successor was proposed. Finally, a candidate was chosen who was unanimously supported by the heads and the Division of Mathematics, and he was confirmed by VAK. Of course, such stands in life brought much trouble to Adian (the lateness in his being elected a member of the Academy was mostly due to his having such a 'high profile') and led to him making many enemies. However, the same strains of character have allowed him to acquire true friends among people of like mind who are impressed by his directness and open temperament. Adian did not wish to recognise government restrictions on human relations as necessary even at a time when this was fraught with various risks.

Even the people closest to Adian would probably not be so bold as to call him an easy person to work with. Everybody who has ever dealt with him knows his adherence to principles and his uncompromising nature with respect to quite diverse questions, as well as his careful attention to details. However, those who have been in closer contact with him (and the authors of the present note belong to this list) well know another aspect. In the end it almost always happens, in some incomprehensible way, that Adian has in fact been right from the very beginning. And his arguments have been at least worth considering always, without exception.

Adian has three adult children, two daughters and one son. His son Ivan graduated from the Faculty of Mechanics and Mathematics at MSU. The older daughter Vera graduated with honours from the Faculty of Philology of MSU and in recent years has taught Russian on contract in London. The younger daughter Lena graduated from the S G Stroganov Moscow State University of Arts and Industrial Design (Ceramics Department). She does painting and ceramic arts and has displayed her works often at the Central House of Artists and at exhibitions of young Russian artists; very recently she was elected a member of the Union of Artists of the Russian Federation.

Article by: *J J O'Connor* and *E F Robertson* based on reference [3] on the recommendation of Lev Beklemishev

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List of References (3 books/articles)

Mathematicians born in the same country

Other Web sites

Sergei Adian (1931-2020)

The word problem and Applications

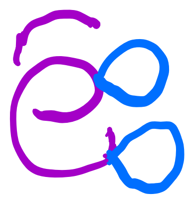
1. Review of covering spaces
2. Free products, free products with amalgamation, HNN Extensions.
3. Bass-Serre theory
4. Presentations of groups, statement of the word problem
5. Dehn function.
6. Examples: Free groups, surface groups, Heisenberg group, Baumslag-Solitar group, Gersten's group and more.
7. Van Kampen's theorem and when fundamental groups don't inject. (Relevant to Adian-Rabin theorem)
8. Turing machines
9. Post's theorem

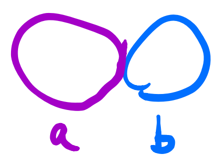
11. Introduction to Van Kampen diagrams and unsolvable word problem (Theorem of Boone-Novlkov)
12. Gromov's theorem on closed geodesics (application to differential geometry)
13. Higman' s theorem (and Universal groups)
14. Adian-Rabin theorem (Markov properties)
15. Homology of groups and Gordon's theorem
16. Hopf exact sequence and Kervaire's theorem.
17. Universal Central extensions and S. Novikov's theorem
18. Other unsolvable topological problems (including non-c.e. Sets);
19. Sobolev homology?

References:

Rotman, Theory of Groups (GTM)
Stillwell, The word problem, Bull AMS
Algorithmic and classification problems in Group theory (Baumslag and Miller, editors, MSRI Publication)
Serre and Kilmer, Trees (Springer) and
Articles by Gersten and Riley on Dehn Functions
Brown, Cohomology of Groups (GTM)
Weinberger, Computers, Rigidity and Moduli (PUP)

$F\langle a, b\rangle$ Covering Spaces.





$\langle b, aba^{-1}, a^2\rangle$

$G * H$ Kurosh Subgroup Theorem

$A *_B C$ $\qquad\qquad$ $A *_B$

Connection to group actions on trees.

$$\langle x, y \mid x^2 = y^2 \rangle$$

$$\langle x, y \mid x^2 = y^2 = e \rangle$$

$$\langle x, y \mid xyx^{-1} = y^3 \quad x^5 = e \rangle$$

$$w = y^{10} x y^2 x^{-1} y^{10}$$

$$\langle x, y \mid xyx^{-1} = y^2 \rangle$$

Word problem.

Problem about groups, not presentations.

Dehn function and geometric meaning of the word problem in terms of isoperimetry and constructing the Cayley graph.

Examples $\mathbb{Z}$

$\mathbb{Z} * \mathbb{Z}$

$\mathbb{Z} \times \mathbb{Z}$ [Van Kempen Diagrams]

$\langle x, y \mid xyx^{-1} = y^2 \rangle$

and worse.

Distortion of subgroups.

VK when not 1-1.

Where we are going:

WP is unsolvable.

Most things can't be told from a presentation (Adian-Rabin).

RE is algebraic notion (Higman).

Geometric + Topological variations.

(+ Analytic?).

Definition. A ***quadruple*** is a 4-tuple of one of the following three types:

$$q_i s_j s_k q_l,$$

$$q_i s_j R q_l,$$

$$q_i s_j L q_l.$$

A ***Turing machine*** T is a finite set of quadruples no two of which have the same first two letters. The ***alphabet*** of T is the set $\{s_0, s_1, \ldots, s_M\}$ of all s-letters occurring in its quadruples.

Definition. An ***instantaneous description*** α is a positive word of the form $\alpha = \sigma q_i \tau$, where σ and τ are s-words and τ is not empty.

"What's on the tape"?" (and "where's the machine")

Definition. Let T be a Turing machine. An ordered pair (α, β) of instantaneous descriptions is a ***basic move*** of T, denoted by

$$\alpha \to \beta,$$

if there are (possibly empty) positive s-words σ and σ' such that one of the following conditions hold:

(i) $\alpha = \sigma q_i s_j \sigma'$ and $\beta = \sigma q_l s_k \sigma'$, where $q_i s_j s_k q_l \in T$;

(ii) $\alpha = \sigma q_i s_j s_k \sigma'$ and $\beta = \sigma s_j q_l s_k \sigma'$, where $q_i s_j R q_l \in T$;

(iii) $\alpha = \sigma q_i s_j$ and $\beta = \sigma s_j q_l s_0$, where $q_i s_j R q_l \in T$;

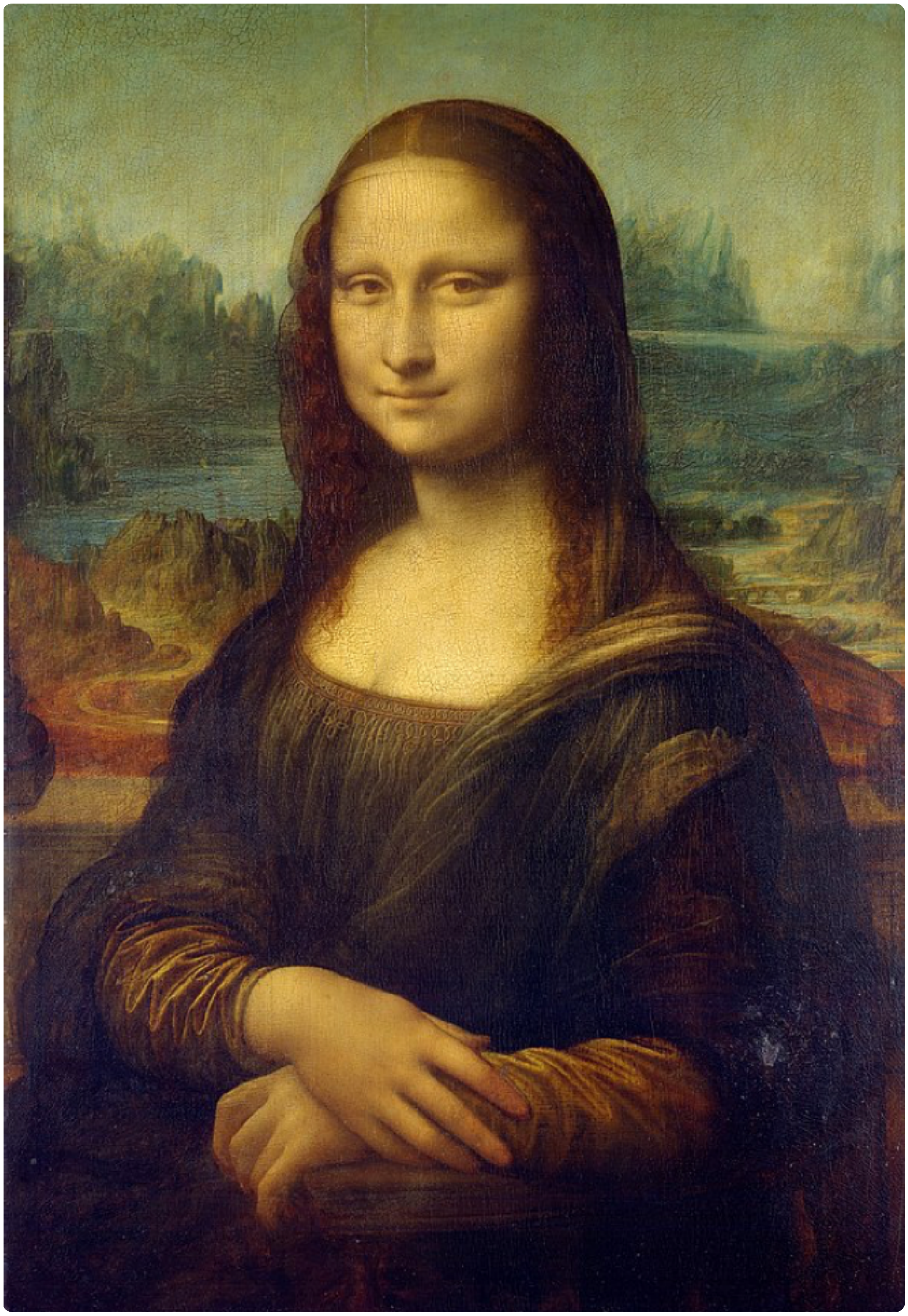
(iv) $\alpha = \sigma s_k q_i s_j \sigma'$ and $\beta = \sigma q_l s_k s_j \sigma'$, where $q_i s_j L q_l \in T$; and

(v) $\alpha = q_i s_j \sigma'$ and $\beta = q_l s_0 s_j \sigma'$, where $q_i s_j L q_l \in T$.

adding blanks at the beginning or ends of tapes.

If α describes the tape at a given time, the state q_i of T, and the symbol s_j being scanned, then β describes the tape, the next state of T, and the symbol being scanned after the machine's next move. The proviso in the definition of a Turing machine that no two quadruples have the same first two symbols means that there is never ambiguity about a machine's next move: if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\beta = \gamma$.

Some further explanation is needed to interpret basic moves of types (iii) and (v). Tapes are finite, but when the machine comes to an end of the tape, the tape is lengthened by adjoining a blank square. Since s_0 means blank, these two rules thus correspond to the case when T is scanning either the last symbol on the tape or the first symbol.



Turing Machine versus Mona Lisa.

Semigroup Word Problem

Suppose that a semigroup Γ has a presentation

$$\Gamma = (X \mid \alpha_j = \beta_j, j \in J).$$

If ω and ω' are positive words on X, then it is easy to see that $\omega = \omega'$ in Γ if and only if there is a finite sequence

$$\omega \equiv \omega_1 \to \omega_2 \to \cdots \to \omega_t \equiv \omega',$$

where $\omega_i \to \omega_{i+1}$ is an ***elementary operation***; that is, either $\omega_i \equiv \sigma\alpha_j\tau$ and $\omega_{i+1} \equiv \sigma\beta_j\tau$ for some j, where σ and τ are positive words on X or $\omega_{i+1} \equiv \sigma\beta_j\tau$ and $\omega_i \equiv \sigma\alpha_j\tau$.

Post's Theorem

Definition. If T is a Turing machine having stopping state q_0, then its ***associated semigroup*** $\Gamma(T)$ has the presentation:

$$\Gamma(T) = (q, h, s_0, s_1, \ldots, s_M, q_0, q_1, \ldots, q_N \mid R(T)),$$

where the relations $R(T)$ are

$$q_i s_j = q_l s_k \qquad \text{if} \quad q_i s_j s_k q_l \in T,$$

for all $\beta = 0, 1, \ldots, M$:

$$\begin{aligned}
q_i s_j s_\beta &= s_j q_l s_\beta && \text{if} \quad q_i s_j R q_l \in T,\\
q_i s_j h &= s_j q_l s_0 h && \text{if} \quad q_i s_j R q_l \in T;\\
s_\beta q_i s_j &= q_l s_\beta s_j && \text{if} \quad q_i s_j L q_l \in T,\\
h q_i s_j &= h q_l s_0 s_j && \text{if} \quad q_i s_j L q_l \in T;
\end{aligned}$$

$$q_0 s_\beta = q_0,$$

$$s_\beta q_0 h = q_0 h,$$

$$h q_0 h = q.$$

Theorem: In $T(\Gamma)$ $\; h q_1 u h = q$

iff u is a stopping state for the machine Γ.

$\Longrightarrow$ is pretty obvious.

$\Longleftarrow$ Focus on "special words $h\alpha h$"

instaneous description

Let ω and ω' be words on $\{s_0, s_1, \ldots, s_M, q_0, q_1, \ldots, q_N\}$ with $\omega \not\equiv q$ and $\omega' \not\equiv q$. If $\omega \to \omega'$ is an elementary operation, then ω is h-special if and only if ω' is h-special.

If $\omega = h\alpha h$ is h-special, $\omega' \not\equiv q$, and $\omega \to \omega'$ is an elementary operation of one of the first five types, then $\omega' \equiv h\beta h$, where either $\alpha \to \beta$ or $\beta \to \alpha$ is a basic move of T.

The last move is to the right

$h\alpha h \;\cdots\; \text{or} \; h\alpha_{k-2}h \; \text{or} \; h\alpha_{k-1}h \; \text{or} \; h\alpha_k h \to q$

but $\exists!$ rightward motion so backmotion is irrelevant: (i.e. is undone by the forward)

$\alpha_{i-1} \leftarrow \alpha_i \rightarrow \alpha_{i+1}$

$\Rightarrow \alpha_{i-1} = \alpha_{i+1}$

<u>Theorem</u>: In $T(\Gamma)$ $h q_1 u h = q$ iff u is a stopping state for the machine Γ.

The rest of the work is modifying $T(\Gamma)$ and arranging for an embedding into a f.p.

$B(T) =$

$\langle$ *generators*: $q, q_0, \ldots, q_N, s_0, \ldots, s_M, r_i, i \in I, x, t, k$ |
relations: for all $i \in I$ and all $\beta = 0, \ldots, M$,

$$\left.\begin{aligned} & x s_\beta = s_\beta x^2, && \Delta_1 \\ & r_i s_\beta = s_\beta x r_i x, && \\ & r_i^{-1} F_i^{\#} q_{i_1} G_i r_i = H_i^{\#} q_{i_2} K_i, && \Delta_2 \\ & t r_i = r_i t, && \\ & t x = x t, && \Delta_3 \end{aligned}\right.$$

$$k r_i = r_i k,$$
$$k x = x k,$$
$$k(q^{-1} t q) = (q^{-1} t q) k. \rangle$$

We won't do all the combinatorics

but will note the various HNN + algems

- $B_0 = \{x \mid \emptyset\} \cong \mathbb{Z}$

- $B_1 = \{B_0, s_0 \dots s_m \mid \Delta_1\}$

  is an HNN extension with s_i all stable letters.

- $B_2 = \{B_1 * Q, r_i \mid \Delta_2\}$

B_2 is HNN of $B_1 * Q$ with r_i as stable letters.

- $B_3 = \{B_2, t \mid \Delta_3\}$

B_3 is HNN of B_2 with t as stable letter

- B is HNN of B_3 with stable letter k.

Lemma 3.6 (Main Technical Lemma). *Let K be a group given by a presentation on a finite or countably infinite set of generators, say*

$$K = < x_1, x_2, \dots \mid R_1 = 1, R_2 = 1, \dots > .$$

For any word w in the given generators of K, let L_w be the group with presentation obtained from the given one for K by adding three new generators a, b, c together with defining relations

$$\begin{array}{llcll}
(1) & a^{-1}ba & = & c^{-1}b^{-1}cbc & \\
(2) & a^{-2}b^{-1}aba^2 & = & c^{-2}b^{-1}cbc^2 & \\
(3) & a^{-3}[w,b]a^3 & = & c^{-3}bc^3 & \\
(4) & a^{-(3+i)}x_iba^{(3+i)} & = & c^{-(3+i)}bc^{(3+i)} & i = 1, 2, \dots
\end{array}$$

$\supset F_\infty \subset F_2\langle b, c\rangle$

where $[w, b]$ is the commutator of w and b. Then

(1) *if $w \neq_K 1$ then K is embedded in L_w by the inclusion map on generators;*

(2) *the normal closure of w in L_w is all of L_w; in particular, if $w =_K 1$ then $L_w \cong 1$, the trivial group;*

(3) *L_w is generated by the two elements b and ca^{-1}.*

If the given presentation of K is finite, then the specified presentation of L_w is also finite.

$$L_w \cong (K * \langle a, b \rangle) * \langle b, c \rangle$$

LHS = RHS

Key Tool is the Van Kampen diagram.

(See e.g. [Riley] or Office Hours with a GEOMETRIC GROUP THEORIST)

First Application to a variational problem

Theorem: Let M be a closed Riemannian manifold with $\pi_1 M$ having unsolvable word problem then M has ∞'ly many closed contractible geodesics.

<u>Remark</u>: (a) $\Longleftrightarrow$ periodic solutions to a certain nonlinear ODE with additional geo properties.

(b) Lusternick - Fet theorem

(c) You are never entitled to contractible ones, that are local minima

of E.(Energy).

These are exactly what

are produced here.

Subgroups of finitely presented groups

By G. Higman, F.R.S.
Department of Mathematics, University of Chicago

(*Received* 20 *February* 1961)

The main theorem of this paper states that a finitely generated group can be embedded in a finitely presented group if and only if it has a recursively enumerable set of defining relations. It follows that every countable Abelian group, and every countable locally finite group can be so embedded; and that there exists a finitely presented group which simultaneously embeds all finitely presented groups. Another corollary of the theorem is the known fact that there exist finitely presented groups with recursively insoluble word problem. A by-product of the proof is a genetic characterization of the recursively enumerable subsets of a suitable effectively enumerable set.

See [Rotman] or [Stillwell]

<u>Defn</u>: P is a Markov property if $0 \notin P$ and $G \in P$, $G \subset K$ $\Rightarrow$ $K \in P$. (e.g. nontrivial, nonabelian, etc.)

THEOREM: (ADIAN - RABIN)

There is no algorithm to tell if a finite presentation describes a property P group.

Proof: Uses HNN contructions

House of Cards.