Here we consider the evolution of a random tree. It starts at time 1 with one node, and at time n, one new node is created and then connected to some earlier node randomly (uniformly).

1. Prove that the average number of vertices with valence = k approaches 2-kn with error o(n) with probability 1. (Can you prove more precise statements?)
2. Let d(v) denote the distance of a node to the initial node. Show that on average d(v) is log(n) and that almost all vertices are at this distance.
3. Show that the average distance of a pair of points is 2logn(n)
4. Show that there is a t>0, so that n1/2 of the vertices are of distance at least (1+t)log(n) away from the initial node, and that the diameter is thus larger than 2log(n)

Consider now a preferential attachment model of a random tree – at stage n, the new node is connected to an earlier vertex either with probability p, entirely at random, or with probability 1-p proportional to its valence.

1. Compute the expected degree sequence. What is the exponent of its power law?

2. What is the expected valence of the initial node at time n?