Review for Prelim # 1

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You should know everything from Chapter 0 except for the section on trigonometry. In particular, it is important that you be well-versed in solving the various kinds of inequalities assigned on the homework. The following exercises will give you an idea of the kind of problems you will be expected to know how to solve:

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Additionally, you should be able to solve quadratic inequalities. Here are some practice problems on quadratic inequalities:

Exercise 1. Solve the following inequalities and express your answers in interval notation:

1. $x^2 - 2x \geq 0$
2. $x^2 - 2x < 0$
3. $x^2 - 4x + 3 > 0$
4. $x^2 + 7x - 3 \leq 0$
5. $x^2 - 4x + 4 \leq 0$
6. $(x - 5649)^{2341291}(x + 23)^{675237} \leq 0$

From chapter 1, you should know the ϵ-δ definition of a limit and be able to use it to prove the existence of limits of linear functions. You should also be able to use the definition of a limit to show that a limit does not exist. The following exercises test this:

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Exercise 2. Prove using an ϵ-δ argument the following limits are as stated

1. $\lim_{x \to 2} 2x - 5 = -1$
2. $\lim_{x \to 0} 2x - 5 = -5$
3. $\lim_{x \to 4} 12x - 9 = 39$

4. $\lim_{x \to -2} x - 3 = -5$
5. $\lim_{x \to 3^+} 3x - 7 = 2$
6. $\lim_{x \to c} 4x - 2 = 4c - 2$

You should know the definition of a continuous function (both at a point and on an interval). You should know Theorem A on pg. 68, Theorem C on pg. 70, Theorem D pg. 72, and Theorem E on pg. 85 and how to use them to evaluate limits and demonstrate the continuity of functions. Finally, you should know the intermediate value theorem and how to use it to find zeros of equations (note that the version of the intermediate value theorem given in the textbook is slightly more general than the one I did in class)

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Exercise 3. Use the limit theorems to prove the following:

1. $\lim_{x \to 1} 3x^4 + 2x^7 + 3 = 8$
2. $3x^4 + 2x^7$ is continuous at every $c \in \mathbb{R}$.
3. $\lim_{x \to 7} \sqrt{x^2 - 13} = 6$
4. $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x - 2} = 1$
5. $\lim_{x \to 3} \dfrac{x^2 - 4x + 4}{x - 2} = 1$
6. $\lim_{x \to 5} \sqrt{\dfrac{(x^2 - 4x - 4)^{11}}{(x^3 + x^2 - 148)^3}} = \dfrac{\sqrt{2}}{4}$
7. Prove that $\frac{x-1}{x-4}$ is continuous at $x = 3$.
8. Prove that $\frac{x-3}{x^2+2x-3}$ is continuous for all $x \neq -1, 3$.
9. Prove that $\sqrt{x-3}$ is continuous at all $x > 3$.
10. Use the intermediate value theorem to show that there is at least one solution to the equation $x^4 + 3x - 3 = 0$.