Math 13100 Section 46 Prelim 2

November 16th, 2007

Instructions

Please show all your work. Calculators, books, and other outside materials are NOT permitted for use on this test. Good luck.

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	Score
Problem 1	15 /15
Problem 2	15 /15
Problem 3	20 /20
Problem 4	15 /15
Problem 5	10 /10
Problem 6	10 /10
Problem 7	15 /15
Total	100

Good job!

Initials sld

1. (a) (5 points) Let $f(x)$ be a differentiable function. Define $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) (10 points)Use your definition to find the derivative of $f(x) = x^2 + x$.

$$f'(x) = \lim_{h \to 0} \frac{((x+h)^2 + (x+h)) - (x^2+x)}{h}$$

$$= \lim_{h \to 0} \frac{(x^2+2xh+h^2) - x^2 + (x+h) - x}{h} \quad \text{(expand)}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + h}{h} \quad \text{(simplify)}$$

$$= \lim_{h \to 0} 2x+h+1 \quad \text{(Theorem C)}$$

$$= 2x+1$$

Initials SlJ

2. Let the position at time t of a particle moving along a horizontal coordinate line be given by $f(t) = t^3 - 9t^2 + 15t$

   (a) (5 points) What is the velocity of the particle at time t? What is the speed of the particle at time t?

   The velocity at time t is the derivative of $f(t)$. Since $f'(t) = 3t^2 - 18t + 15$, the velocity, $v(t)$, at time t is $\boxed{v(t) = 3t^2 - 18t + 15}$

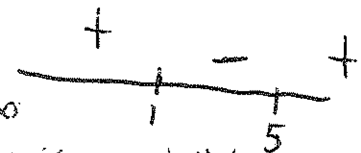
   Since speed is the absolute value of velocity, the speed, denoted by $s(t)$, at time t is $\boxed{s(t) = |3t^2 - 18t + 15t|}$

   (b) (5 points) At what times is the particle moving to the right? At what times is it moving to the left?

   The particle is moving to the right when the velocity is positive. and the left when the velocity is negative.

   We need to solve $v(t) > 0$

   $3t^2 - 18t + 15 > 0 \iff 3(x-1)(x-5) > 0$



   The particle is moving to the right when $t \in (-\infty, 1) \cup (5, \infty)$

   " " " the left when $t \in (1, 5)$

   (c) (5 points) When is the speed of the particle increasing?

   The speed is increasing when the particle is accelerating in the same direction as it is moving. The acceleration is $a(t) = 6t - 18$. The is positive for $t > 3$ and negative otherwise.

   We conclude the speed is increasing when $t \in (1, 3)$ and $t \in (5, \infty)$

Initials Sld

3. (20 points) Use the rules of differentiation to calculate $f'(x)$ when $f(x)$ is the following:

   (a) $f(x) = x^3 + 4x^2 + 3x + 7 + 1/x$

   $f'(x) = 3x^2 + 8x + 3 - \frac{1}{x^2}$ (Power rule)

   (b) $f(x) = (2 + g(x))^{30}$ where $g(x)$ is a differentiable function.

   ~~B~~ Using the chain and power rule, we see that

   $f'(x) = 30(2+g(x))^{29} g'(x)$

   (c) $f(x) = \frac{3x+9}{x^3+1}$

   By the quotient rule,

   $$f'(x) = \frac{(x^3+1)3 - (3x+9)(3x^2)}{(x^3+1)^2}$$

   (d) $f(x) = x^2\sqrt{x+1}$

   Using the product rule, we have

   $f'(x) = 2x\sqrt{x+1} + x^2 \, 1/2 \, (x+1)^{-1/2}$

Initials SId

4. (a) (10 points) Consider the equation $x^2y + 2 = y^2x$. Use implicit differentiation to find $\frac{dy}{dx}$.

If we pretend $y = f(x)$ for some differentiable function $f(x)$, we get the equation

$$x^2 f(x) + 2 = f(x)^2 x.$$

If we differentiate this, we get

$$x^2 f'(x) + 2x f(x) = f(x)^2 + x\, 2 f(x) f'(x)$$

Rearrange terms and

$$f'(x)(x^2 - 2f(x)x) = f(x)^2 - 2x f(x) \Rightarrow f'(x) = \frac{f(x)^2 - 2x f(x)}{x^2 - 2f(x)x}$$

If we substitute $\frac{dy}{dx}$ for $f'(x)$ and y for $f(x)$ we see that

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2yx}$$

(b) (5 points) Find the tangent line to the graph of $x^2y + 2 = y^2x$ at the point $(1, 2)$

Using $\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2yx}$ we see that at $(1,2)$

the slope of the tangent line is $\frac{4-4}{-3} = 0$

Since it goes through $(1,2)$, the equation is

$$(y-2) = 0\,(x-1) \Rightarrow y = 2$$

Initials SID

5. (a) (5 points) If $f(x)$ is differentiable at a point c, is it necessarily true that $f(x)$ is continuous at c?

Yes.

(b) (5 points) If $f(x)$ is continuous at a point c, is it necessarily true that $f(x)$ is differentiable at c? Explain your answer.

No. The function $f(x) = |x|$ is continuous at $x=0$,
but is not differentiable at $x=0$

Initials SID

6. (10 points) The edge of a cube is increasing at the rate of 4 inches per second. How fast is the volume of the cube increasing when an edge is 10 inches long? (Hint: The volume of a cube with edge length e is e^3)

Let $e(t)$ be the length of an edge of the cube at time t. The volume at time t of the cube is

$V(t) = e(t)^3$ (1)

We want to find $\frac{dV}{dt}$ when $e(t) = 10$.

If we differentiate equation 1, we see

$$\frac{dV}{dt} = 3\,e(t)^2 \frac{de}{dt}$$

If we substitute $e(t) = 10$ and $\frac{de}{dt} = 4$ into this equation we get

$$\frac{dV}{dt} = 3 \cdot 10^2 \cdot 4 = 1200 \ in^3/sec$$

Initials $5|0$

7. (a) (10 points) Find the tangent line to $f(x) = \sqrt{x}$ at $x = 25$.

First we compute that $f'(x) = 1/2\, x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

The slope of the tangent line to $\sqrt{x}$ at $x=25$ is $f'(25) = \dfrac{1}{2\sqrt{25}} = 1/10$.

Since the tangent line goes through the point $(25, f(25)) = (25, 5)$ the equation for the tangent line is

$$(y-5) = \frac{1}{10}(x-25) \iff y = \frac{1}{10}x + 5/2$$

(b) (5 points) Use the tangent line you found in the first part of this problem to approximate $\sqrt{26}$.

We will approximate $\sqrt{26}$ by the tangent line above.

$$\sqrt{26} \approx \frac{1}{10}\, 26 + 5/2 = 5.1$$