Standing at the board on the first day of the quarter, I have a completely different idea of what mathematics is than my calculus students. Though very bright and hardworking, by and large my calculus students have never really ‘encountered’ mathematics. They’ve taken high school courses in algebra, geometry, and possibly even calculus, but these classes generally focused on memorization and computation. As a mathematician, I understand the field to be more akin to a creative discipline than a science. Beyond guiding my students to a comprehensive understanding of the subject at hand, my goal as an instructor is to teach my students how to ‘think mathematically’: how to connect apparently different ideas, by looking for patterns and parallels between the situations; how to move between different levels of abstraction; and how to evaluate arguments for rigor and validity. I work to make my classroom a ‘mathematical laboratory’, a space where my students ask questions and engage in an active discussion about what they are learning (and why), where my students work together, sharing ideas and differing strengths. I want my students to take away an appreciation for why this perspective and method can help them learn.

In teaching a service course such as calculus, as the term suggests, there is requisite material to cover. Instructors in other departments and courses have expectations for what a student who has passed my course understands. But my second set of goals is not at odds with this purpose: the mathematical tools I impart assist students with learning the material of the course as well as future math courses, and also build the basis for an informed, analytical, and skeptical approach to the world in general; in a word, these tools are an integral aspect of a student’s education. Moreover, by treating the course as a creative venture, I see my challenge is to spark curiosity in my students. A curious student is a motivated learner; a curious student will work hard to understand the material, as opposed to a student who just memorizes algorithms and formulae.

How, then, to foster this curious student? Especially with a classroom of students taking my course as a requirement, students are often working against math anxiety and a belief that they are inherently ‘bad’ at math. This is especially true among women, students of racial or ethnic minority background, and students with a weaker mathematical background. I take my first step toward encouraging curiosity within a student by breaking down their idea of what ‘math’ means, in order to grow a student’s confidence in their own ideas. On the first day of class, I ask students to work together in small groups to solve a mathematical puzzle. This will generally be unlike anything they’ve been ask to do in a math course before, but finding a solution requires them to think and work together like mathematicians: they work through examples, guess answers and try them out, test each other’s ideas. In this process, they’ll make mistakes, but importantly, they’ll see their peers also making errors, and together they’ll find these errors and fix them. Mistakes become a part of the learning process. As the barriers of anxiety and fear are broken down, a mathematical question is no longer a monster demanding sacrifice, but rather a friend, beckoning the student to come explore and experiment with them.

When I am teaching a course which requires me to move quickly through the material, I seek ways to make a lecture inviting and conceptual. This is not to say I avoid examples - on the contrary, I emphasize examples as a way of capturing the essential idea of a topic. For example, when introducing the idea of continuity, I focus on helping my students understand the ways a function can fail to be continuous, highlighting essential similarities and differences between different types
of discontinuities. By giving these examples the extra structure of this comparison, it is no longer a list to memorize, but a key to unlock the concept of continuity.

This presents a challenge, however: students need to be able to take this core idea or picture, abstract it to a broader framework for understanding the concept at hand, and reapply it to solve specific problems. I design homework to be a low stakes way of practicing this skill. While much of the homework I assign is from the textbook, I also give a couple problems each week which help build this muscle. After giving my example-based introduction to continuity, I ask my students to compare these examples of discontinuities with the formal definition of continuity, applied to a specific function.

As a method for my student and myself to check in on our understanding of the material, I give weekly, five minute quizzes. These are ungraded, but I read through them to get a sense of problem areas I should spend more time on. After the quizzes are handed in, I quickly go over the problem, so students can also see whether they are understanding the material.

Much of what I’ve described here is with a calculus student in mind, but many of these principles still apply in the context of teaching courses to math majors. My standards are higher in this context, but learning is still motivated by curiosity and occurs primarily outside of the classroom. In this context, I take the challenge of teaching my students to think like mathematicians one step further: I ask them to complete an independent project on their own or in small groups, over the course of the term. This gives students the opportunity to learn independently, digest and interpret ideas, and practice reformulating and explaining the topic clearly in writing and in an oral presentation.