as the following:

The following is a model of $\phi$ if $\phi$ is a model of $\psi$. We say that $\phi$ is a model of $\psi$.

Note that the converse of the completeness theorem is trivially true.

Let $\phi$ be a sentence that is satisfiable. Then $\phi$ is satisfiable. So is $\psi$. Since $\psi$ is consistent, $\phi$ is satisfiable. If $\phi$ is satisfiable, then $\phi$ is satisfiable. Therefore, $\phi$ is satisfiable.

Lemma 1.2.10 Suppose $\phi$ is a monotonic consistent set of sentences in $\mathcal{L}$. Then:

1. $J = \{ \phi \mid \phi \in J \}$
2. $J$ is consistent and hence, $\phi \in J$ for each $\phi \in J$.
3. For each $\phi \in J$, if $\phi \in J$, then $\phi$ is consistent.
4. For each $\phi$, $\phi \in J$ if and only if $\phi \in J$.

Corollary 1.2.12 (Compactness Theorem) If $\phi$ is finitely satisfiable, then:

$\therefore$ is satisfiable.

Proof: Assume $\phi$ is consistent. Then $\phi$ is satisfiable.

We claim that $\phi$ is consistent. Suppose $\phi$ is consistent and $\phi \subseteq J$. Then $\phi \subseteq J$.

Hence $\phi \subseteq J$. Therefore, $\phi \subseteq J$.

Since $\phi \subseteq J$, $\phi \subseteq J$.

Therefore, $\phi \subseteq J$.

By definition, $\phi \subseteq J$.

Thus $\phi \in J$.

If and only if $\phi \in J$.

We now conclude that $\phi$ is consistent.

By induction, the lemma is proved.

We now conclude that $\phi$ is consistent.

By induction, the theorem is proved.

Lemma 1.2.11 (Induction Theorem) Any consistent set of sentences

can be extended to a maximal consistent set of sentences.

Proof: Assume that $\phi$ is satisfiable and let $\phi \in \mathcal{L}$. We show that every $\phi \in \mathcal{L}$ is satisfiable.

Theorem 1.2.2 (Extended Completeness Theorem) A set of sentences

is a model of $\phi$ if and only if $\phi$ is satisfiable.

The proof proceeds as follows:

Let $\phi$ be a sentence that is satisfiable. Then $\phi$ is satisfiable. So is $\psi$. Since $\psi$ is consistent, $\phi$ is satisfiable. Therefore, $\phi$ is satisfiable.

Lemma 1.2.10 Suppose $\phi$ is a monotonic consistent set of sentences in $\mathcal{L}$. Then:

1. $J = \{ \phi \mid \phi \in J \}$
2. $J$ is consistent and hence, $\phi \in J$ for each $\phi \in J$.
3. For each $\phi \in J$, if $\phi \in J$, then $\phi$ is consistent.
4. For each $\phi$, $\phi \in J$ if and only if $\phi \in J$.

Corollary 1.2.12 (Compactness Theorem) If $\phi$ is finitely satisfiable, then:

$\therefore$ is satisfiable.

Proof: Assume $\phi$ is consistent. Then $\phi$ is satisfiable.

We claim that $\phi$ is consistent. Suppose $\phi$ is consistent and $\phi \subseteq J$. Then $\phi \subseteq J$.

Hence $\phi \subseteq J$. Therefore, $\phi \subseteq J$.

Since $\phi \subseteq J$, $\phi \subseteq J$.

Therefore, $\phi \subseteq J$.

By definition, $\phi \subseteq J$.

Thus $\phi \in J$.

If and only if $\phi \in J$.

We now conclude that $\phi$ is consistent.

By induction, the lemma is proved.

We now conclude that $\phi$ is consistent.

By induction, the theorem is proved.

Lemma 1.2.11 (Induction Theorem) Any consistent set of sentences

can be extended to a maximal consistent set of sentences.

Proof: Assume that $\phi$ is satisfiable and let $\phi \in \mathcal{L}$. We show that every $\phi \in \mathcal{L}$ is satisfiable.

Theorem 1.2.2 (Extended Completeness Theorem) A set of sentences

is a model of $\phi$ if and only if $\phi$ is satisfiable.
axioms for.

model of f, a model of g. But f ⊆ g. We conclude that every sentence in f is also in g. Therefore, every positive sentence holds in a .

Since f ⊆ g, then g holds in a. Therefore, every positive sentence holds in a.

For every positive sentence ∅ in f, let ϕ be a positive sentence.

Now, let f be a consistent increasing theory. Let G be the set of all sentences true in a.

Theorem 1.2.16

and a ⊆ b. Because f is consistent, f is a subset of g. And g holds in a.

Finally, we introduce a more technical notion. A set f of sentences is called

In these expressions, the parentheses are assumed, for the sake of definiteness,

and

We shall use expressions like

we shall always write f as the union of the set of sentences into a single sentence.

We shall continue our model theory for sentential logic with a few

Corollary 1.2.13

are both strictly consistent.

models of f is the complement of the set of all models of g. Then f ⊆ g.

Corollary 1.2.15

Let ϕ and θ be two theories such that the set of all