(1) Show that $K_{(m,n)} = \{ e | \varphi_e(m) = n \}$ is not computable.

(2) If $A, B \subseteq \mathbb{N}$ are sets then we say $A \leq_m B$ if there’s a computable function $f$ such that $f(A) \subseteq B$ and $f(\overline{A}) \subseteq \overline{B}$. If $f$ is 1-1 say that $A \leq_1 B$.

   (a) Show that if $B$ is c.e. and $A \leq_m B$ then $A$ is c.e.
   (b) Show that if $\overline{K} \leq_m A$, then $A$ isn’t c.e.
   (c) Use the above to show that Inf isn’t c.e. (In fact Inf contains a lot more information than $K$).

(3) If $I \subset \mathbb{N}$ is a set such that whenever $e \in I$ and $\varphi_i = \varphi_e$, $i \in I$ too, say that $I$ is an index set; show that if $\emptyset \subsetneq I \subsetneq \mathbb{N}$ is an index set, then $K \leq_1 I$ or $\overline{K} \leq_1 I$. From the previous question, we now know that $I$ is not computable. Hint: choose an index $e$ for which $\varphi_e$ has empty domain, and an index $i$ so that exactly one of $\varphi_e, \varphi_i$ is in $I$.

(4) Let $\mathcal{L}$ be a finite language. Give a reasonably complete sketch of a proof that the set of validities $\{ \varphi : \varphi$ is a wff and $\models \varphi \}$ is c.e.

   [“Reasonably complete sketch” means that a key step involves checking membership in each of Enderton’s six classes of Logical Axioms: allocate a sentence or two to each class.]

(5) Let $\mathcal{L}$ be finite, $\Gamma$ be a set of axioms and $T = Cn(\Gamma)$. Show that $T$ is c.e.