(1) Let $\kappa_\beta$ be an infinite cardinal. Prove that if $\beta$ is a limit ordinal, then $\text{cof}(\kappa_\beta) = \text{cof}(\beta)$. Prove that if $\beta$ is a successor ordinal, then $\kappa_\beta$ is regular. Show by example that there are arbitrarily large cardinals with countable cofinality.

(2) Verify that the following is a well-ordering of the class Ord $\times$ Ord.

$$\text{max}(\alpha, \beta) < \text{max}(\gamma, \delta) \iff \begin{cases} (\alpha, \beta) < (\gamma, \delta) \lor (\alpha, \beta) = (\gamma, \delta) \land \alpha < \gamma) \lor (\alpha, \beta) = (\gamma, \delta) \land \alpha = \gamma \land \beta < \delta) \end{cases}$$

(3) For $\alpha$ an ordinal, define $W(\alpha)$ as follows: (i) $W(0) = 0$; (ii) $W(\alpha + 1) = P(W(\alpha))$; for $\alpha$ a limit ordinal, (iii) $W(\alpha) = \bigcup_{\beta<\alpha} W(\beta)$, where $P$ denotes power set. Using transfinite induction, show that $W(\alpha)$ is transitive for each $\alpha$.

(4) In the notation of the previous problem, let $M = (W(\omega), \epsilon)$ be the structure whose domain consists of the elements of $W(\omega)$ and where $\epsilon$ has the usual meaning. Which axioms of ZFC does $M$ satisfy?

(5) Let $M$ be a model, in any language, whose domain is countable. Prove that the following are equivalent:

(a) $M$ has uncountably many automorphisms.

(b) If $B$ is a finite subset of the domain of $M$ then there is a non-identity automorphism of $M$ which is the identity when restricted to $B$. 

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