THE UNIVERSITY OF CHICAGO
2002 Summer VIGRE Program for Undergraduates

This announcement describes an eight-week summer program of research and teaching for undergraduates at the University of Chicago. Its first year of operation was 2000, and it will continue for at least the next three years.

In this program, students have the opportunity for study and research in mathematics together with work in two of the outreach programs of the Department of Mathematics. Students participate in one or two courses taught by Department of Mathematics faculty members. They also work as counselors in the Young Scholars Program (YSP) and SESAME teacher development program.

The purpose of the summer VIGRE program is to provide an opportunity for students to be involved in a deeper experience in mathematics than is usually available during the academic quarters and to allow them to be effective partners in the educational outreach programs of the Mathematics Department. This program is especially beneficial for undergraduates who are considering graduate study and research in mathematics and for those who are interested in teaching mathematics at any level.

DATES: June 17–August 9, 2002

STIPENDS: Each student will receive a stipend of $3000

APPLICATIONS: Students must be currently registered students at the University of Chicago and must be United States citizens or permanent residents. Application forms for the summer of 2002 are now available in Eckhart 211 and 212 and are due March 8, 2002. Completed applications should be returned to Ryerson 350.

THE PROGRAM OF STUDY AND RESEARCH: Students attend one, two, or three courses taught by Department of Mathematics faculty. These courses consist of lectures and problem solving sessions; graduate student assistants run help and problem sessions. Some research problems and some problems aimed simply to aid understanding are introduced by the professors. No previous knowledge or study in the areas taught is required. In addition, opportunities for reading and research with graduate students and/or faculty are offered.

STUDY AND RESEARCH SCHEDULE: The first two weeks have a larger proportion of lectures than the remaining six, the aim being to give enough background to allow students to work in groups or alone later in the program. The program is "frontloaded" with problems to be presentedby Babai and Sally that can be worked on throughout the program — and later!! There are afternoon talks and problem sessions in the third through sixth weeks. There is a more flexible and informal program in the last two weeks. Students are encouraged to work together and to organize evening sessions. Graduate student assistants are on hand during the problem sessions and are available for tutorials.

COURSE OFFERINGS: The following courses will be offered. They range widely in aims and scope, but all are designed to be as self-contained as possible, minimizing prerequisites and maximizing mathematical depth. While problem sessions need not be limited to the weeks listed below, the faculty lectures will be spread over the eight weeks of the program as indicated in the description and summarized in the calendar that follows. Advanced students who wish to engage in "reading and research" should consult with Professor May, who will arrange suitable matchups.

THE LIST OF COURSES

1. RATIONAL APPROXIMATIONS OF IRRATIONAL NUMBERS (weeks 1–4)
Paul Sally
In this seminar, we will study various aspects of rational approximation of irrational numbers. The material begins at the level of the Pigeon Hole Principle and the Mean Value Theorem for functions of one variable and proceeds through some of the impressive developments in this area over the past 150 years. The discussion will be based on the book "Exploring the Number Jungle: a Journey in Diophantine Analysis", by Edward Burger. Quoting from a review:

> "[This book] is designed as a textbook at the undergraduate level, with lots of exercises. The choice of material is very nice: Diophantine approximation is the underlying theme, but the tour has side trips to elliptic curves, Riemann surfaces, and p-adic analysis. The writing style is relaxed and pleasant. But none of the theorems is proved; in all cases, the proofs are left as exercises. Numerous hints are given, and the devoted student should be able to use them to get through the material."

We will go through as much of this book as possible with the students in the seminars providing the proofs.
PQ Math 161-2-3

2. DISCRETE MATHEMATICS (weeks 1–4)
Laszlo Babai
The course covers topics in number theory, discrete Fourier transform, combinatorial structures, linear algebra and discrete probability, highlighting surprising interactions between these areas. Students will discover the field through solving sequences of challenging problems. A number of open problems will also be discussed. Returning students will not be bored!
PQ: CS-174 (Discrete Math) or CS-270 (Algorithms) helpful. Basic linear algebra and finite fields desirable. Interested students are strongly encouraged to take Math-284, a.k.a. CS-274, Honors Combinatorics and Probability, offered in Spring, see http://www.cs.uchicago.edu/courses/descriptions.php.

3. DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES (weeks 1–8)
Benson Farb (1–2), Peter May (3–4), Mel Rothenberg (5–6), Jeffrey Brock (7–8)
Topics will include: geodesic curvature, torsion of curves in R^3, Milnor's famous theorem (proved as a freshman!!) that a knotted curve must have geodesic curvature $> 4\pi$, basic surface theory, the Gauss map, Gaussian and mean curvature of a surface, Gauss's classic "Theorem Egregium", the length curvature formula, minimal surfaces and soap bubbles, and geodesic curvature in non-Euclidean geometry. The discussion will be based in part on the book "Elementary Differential Geometry" by Barrett O'Neill. Here is a quote from a review:

> "This book on elementary differential geometry can be highly recommended. Its emphasis is on the geometric content of the subjects treated, whereas it takes great pains to be formal, precise and up-to-date in terminology. The details are fully exposed, and there is no "leaving to the reader" as is often found in books on differential

geometry because of the inherent difficulty of the subject. Therefore, the book should also be an excellent help for those who want to learn the subject from the text only."

PQ: Math 161-2-3. Some multvariable calculus would be helpful but will be reviewed as needed.

4. INTRODUCTION TO MATHEMATICAL QUANTUM MECHANICS (weeks 5–8)
Peter Constantin and Alexander Kiselev
Topics:
Hamiltonian Systems Basic Principles of Quantum Mechanics
Simplest Models of Quantum Mechanics:
Harmonic Oscillator
Free Particle on a Line
Spectral Theorem
Discrete Schrödinger Equation
Classification of Spectra and Quantum Dynamics
The course is intended as an introduction to mathematical quantum mechanics. We will cover basic principles and discuss a variety of simple, yet interesting quantum systems. Some of the techniques that we are going to develop, such as the stationary phase method or Hausdorff dimensions are also useful in many areas of mathematics outside quantum mechanics. Although it is our intention is to keep the course elementary, we hope to come very close to topics of current research interest.
PQ Math 161-2-3, 203-4-5, or 207-8-9;
Math 254-5-6 or 257-8-9 is desirable.

5. SOME CLASSICAL INVARIANT THEORY (weeks 5–6)
Bob Kottwitz
The course will give a gentle introduction to some topics in classical invariant theory, e.g. the invariants of the binary quartic.
PQ Math 161-2-3

6. CATEGORIFICATION AND DECATEGORIFICATION (weeks 7–8)
Peter May
Alias: mystification and demystification
A philosophical discussion of new ways of "thinking mathematics", describing how categorical language allows the generation of new mathematics. To illustrate, we will discuss Atiyah's rigorous and elementary axiomatization of a "Topological Quantum Field Theory". While the axioms can well be assimilated on an undergraduate level, the proofs that TQFT's satisfying the axioms actually exist in dimensions ≤ 3 is some of the deepest 20th century mathematics. The proofs in dimension 4 are in gestation, and dimensions ≥ 5 are terra incognito.
PQ Curiosity and a stomach for abstraction; course 3 (Geometry) will be relevant.

	June 17 – 28	July 1 – 12	July 15 – 26	July 29 – Aug 9
Approximations	Sally	Sally		
Discrete	Babai	Babai		
Geometry	Farb	May	Rothenberg	Brock
Quantum mechanics			Constantin	and Kiselev
Invariant Theory			Kottwitz	
Categorification				May

THE COUNSELOR PROGRAM: VIGRE students also serve as counselors in YSP and SESAME. YSP is a four-week program for mathematically talented seventh through twelfth graders. There are three components: one for students in grades 7-8, one for students in grades 9-10, and one for students in grades 11-12. The YSP consists of lectures, problem solving sessions led by counselors, and computer sessions. VIGRE counselors are assigned to a particular component and to a small group of students for problem solving and computer sessions. SESAME is a two-week program for elementary teachers from the Chicago Public Schools. VIGRE Counselors work in one of several courses in the SESAME program, and serve in much the same capacity as they do in YSP.

WEEKLY YSP and SESAME WORK SCHEDULE:
Week 1 (June 17 – 21) A few orientation meetings
Week 2 (June 24 – 28) YSP training sessions (mornings only)
Week 3 (July 1 – 5) YSP duties 9 a.m. - 2:30 p.m.
Week 4 (July 8 – 12) YSP duties 9 a.m. - 2:30 p.m.
Week 5 (July 15 – 19) YSP duties 9 a.m. - 2:30 p.m.
Week 6 (July 22 – 26) YSP duties 9 a.m. - 2:30 p.m.
Week 7 (July 29 – August 2) SESAME duties either morning or afternoon
Week 8 (August 5 – 9) SESAME duties either morning or afternoon