PROGRAM NOTES AND ABSTRACTS FOR WEEKS 5 AND 6

Apprentice Program: Daniil Rudenko

Continuing We will cover a variety of topics in algebra, geometry and combinatorics.

Continuous combinatorics: Alexander Razbarov

TITLE CONTINUOUS COMBINATORICS

ABSTRACT: Combinatorics was conceived, and then developed over centuries as a discipline about finite structures. In the modern world, however, its applications increasingly pertain to structures that, although finite, are extremely large: the Internet network, social networks, statistical physics, to name just a few. And the numerical characteristics researchers are normally interested in are "continuous" in the sense that small perturbations in the structure do not change the output very much. This makes it very natural to try to think of the "limit theory" of such objects by pretending that "very large" actually means "infinite". It turns out that this mathematical abstraction is very useful and instructive and leads to unexpected connections with many other things, both in mathematics and computer science.

This mini-series of lectures is intended to be a light and informal introduction to the area; we will go as far as the time permits.

Probability: Greg Lawler and Mark Cerenzia

TITLE: Random Walk and the Heat Equation

ABSTRACT: I will give a long series of lectures (at least three weeks) focusing on the closely related topics of random walk, harmonic functions, and diffusion of heat. I will start with the discrete case where understanding long range behavior boils down to a question in linear algebra. Along the way I will prove some important facts: Stirling?s formula and the (local) central limit theorem.

Next I will do the continuous time analogue which brings in Brownian motion, the usual heat equation from PDE, and show how the construction in the discrete case leads to Fourier series. I expect these ideas will be enough for the first two weeks and I will decide later (with possible participant input) about what to do later.

The only (unofficial!) prerequisites for these lecture are linear algebra and "undergraduate analysis" which is sometimes called "advanced calculus". At Chicago, this means Math 20250 and 20300. Much of the material that I will discuss appears in a small book of the same title published by AMS Press. For copyright reasons, I am not able to hand out copies but a near finished draft of the book is still available on the Web and can probably be found with a Google search.

Analysis: Guher Camliyurt

TITLE (Guher Camilyurt, continuing): First order equations in PDEs

ABSTRACT: In this series of talks, we will mainly discuss first order partial differential equations. First, we will consider the transport equation and briefly introduce the method of characteristics. In the second lecture, we will obtain a general existence and uniqueness result for a period of time, which will lead to examples of singularities. In the last lecture, we will talk about the traffic flow problem to give some ideas about conservation laws.

Number Theory: Akhil Mathew

TITLE (Mathew, continuing): Introduction to quadratic forms over fields

ABSTRACT: I will give an introduction to the theory of quadratic forms over fields, leading up to the Hasse-Minkowski theorem that classifies quadratic forms over the rational numbers.

Special talk. Category theory: Eugenia Cheng

TITLE: Associativity, Commutativity and Units: a Higher-dimensional ballet

Associativity, commutativity and unit laws are axioms we typically learn about early on, in the context of numbers. We might then take them for granted until we meet non-commutative situations, such as multiplication of matrices, or symmetry groups. In higher dimensions we start to encounter non-associativity and non-unitality as well, but there is more nuance: rather than associativity simply being true or not true, there are shades of gray, where associativity holds up to isomorphism, equivalence, or just some sort of map. In this talk I will describe how those familiar three families of axioms become the essence of all the interesting features of weak higher-dimensional category theory. Moreover, rather than being three different types of axioms they are inextricably related via a higher-dimensional version of distributivity. The ballet they present is one of ebb and flow, give and take, where rigidity for one ?dancer? always needs to be offset by flexibility in another. I will show that the apparently mundane math of high school has deep category theoretical insights embedded in it, if we care to look. I will not assume any prior knowledge of category theory.

Algebraic Topology: Peter May and others

Continuing: A guess is that we will introduce stable homotopy theory and maybe describe homotopical analogues of the monadicity theorem that are very important in algebraic topology. At some point I hope to explain how in principle the classifying spaces of symmetric groups "determine" the stable homotopy groups of spheres.

TITLE: Coambiguous concepts

ABSTRACT: We will start with some categorical linguistics, introducing the categorical language required for comparisons between subjects and comparisons between concepts. "Coambiguous concepts" are different definitions that non-obviously have the same or equivalent content. The idea leads to seriously interesting comparisons, either relating different areas of mathematics or giving alternative perspectives on a single area. The first week, we will introduce the categorical language that allows us to make such comparisons. Don't be put off by the undefined terms below!

Example: Finite T_0 topological spaces and finite posets are isomorphic concepts By contrast, what do we mean by equivalent concepts?

Example: Finite dimensional real vector spaces and their maps are equivalent to finite real matrices and matrix multiplication

Example: Finite sets are equivalent to the set of sets $\mathbf{n} = \{1, \cdots, n\}$

Example: Groups are equivalent to free groups modulo relations

Generalization next week: Monads and their algebras; the categorical "monadicity theorem".

TITLE: Homotopically coambiguous definitions

More deeply, what do we mean by concepts that are homotopically coambiguous, by which we mean that different definitions give homotopically equivalent concepts?

Example: Topological spaces, simplicial sets, small categories, and posets are homotopically equivalent concepts.

Example: Simplicial abelian groups and chain complexes are homotopically equivalent concepts.

Possible topics to be explored in more depth later on

Topic 1: Finite spaces and larger contexts (book in progress)

Topic 2: An introduction to stable homotopy theory and spectra

Slide talk: `https://www.youtube.com/watch?v=vRsrCNLkSA0`

Topic 3: An introduction to equivariant homotopy and cohomology theory

Slide talk: `https://www.cornell.edu/video/peter-may-equivariant-cohomology`

Topic 4: Operads and their algebras

Old new example: n-connective spaces are homotopically equivalent to "monadically coequalized" n-fold suspensions of E_n-spaces.

Old new example: Connective spectra are homotopically equivalent to "monadically coequalized" suspensions of E_∞-spaces.