PROGRAM NOTES AND ABSTRACTS FOR WEEKS 3 AND 4

Apprentice Program: Daniil Rudenko

Continuting We will cover a variety of topics in algebra, geometry and combinatorics.

Probability: Greg Lawler

TITLE: Random Walk and the Heat Equation

ABSTRACT: I will give a long series of lectures (at least three weeks) focusing on the closely related topics of random walk, harmonic functions, and diffusion of heat. I will start with the discrete case where understanding long range behavior boils down to a question in linear algebra. Along the way I will prove some important facts: Stirling?s formula and the (local) central limit theorem.

Next I will do the continuous time analogue which brings in Brownian motion, the usual heat equation from PDE, and show how the construction in the discrete case leads to Fourier series. I expect these ideas will be enough for the first two weeks and I will decide later (with possible participant input) about what to do later.

The only (unofficial!) prerequisites for these lecture are linear algebra and "undergraduate analysis" which is sometimes called "advanced calculus". At Chicago, this means Math 20250 and 20300. Much of the material that I will discuss appears in a small book of the same title published by AMS Press. For copyright reasons, I am not able to hand out copies but a near finished draft of the book is still available on the Web and can probably be found with a Google search.

Analysis: Beniada Shabani and Guher Camliyurt

TITLE (Beniada Shabani, continuing): Hamilton-Jacobi equations

ABSTRACT: Hamilton-Jacobi equations are first order differential equations that arise naturally in optimal control theory, finance, game theory, physics, imaging, etc. In the first talk we will formally derive the equations from a deterministic control problem, using Bellman's dynamic programming principle. Next we describe a different approach to Hamilton-Jacobi equations in terms of calculus of variations and derive the Hopf-Lax formula for the solution of the problem. The final part will be dedicated to a weaker notion of solutions called the viscosity solutions. A Calculus sequence should suffice for understanding most of the concepts.

TITLE (Guher Camilyurt): First order equations in PDEs

ABSTRACT: In this series of talks, we will mainly discuss first order partial differential equations. First, we will consider the transport equation and briefly introduce the method of characteristics. In the second lecture, we will obtain a general existence and uniqueness result for a period of time, which will lead to examples of singularities. In the last lecture, we will talk about the traffic flow problem to give some ideas about conservation laws.

Number Theory: Matthew Emerton, Akhil Mathew

TITLE (Emerton): Computing special values of zeta and L-functions

ABSTRACT: I will explain how to compute values of the zeta function, and of L-functions, especially at points where the series diverges.

TITLE (Mathew): Introduction to quadratic forms over fields

ABSTRACT: I will give an introduction to the theory of quadratic forms over fields, leading up to the Hasse-Minkowski theorem that classifies quadratic forms over the rational numbers.

Dynamical systems: Davi Obata and Wenyu Pan

TITLE: Introduction to dynamical systems through examples

ABSTRACT (Weeks 3 and 4): Our goal is to give an introduction to dynamical systems with a few examples. In this first week, we will study rotations and multiplication by a constant on the circle. We will study a few topological and ergodic properties of these systems.

In the second week, we will introduce the Gauss map on $(0, 1)$, a measure-preserving transformation, to study continued fractions. We will discuss how to use the ergodicity of the Gauss map to study the approximation of irrational numbers by rational numbers.

Representation theory: Justin Campbell

TITLE: Representation theory and Weil's Rosetta Stone

ABSTRACT: Representation theory is a broad web of interconnected results and heuristics, which both undergirds and bridges between fields as seemingly disparate as number theory, algebraic geometry, and quantum mechanics. In these talks I will attempt to convey some of the flavor of this vast subject, emphasizing the unifying perspective it affords.

Special talk (July 5): Shmuel Weinberger

TITLE: Topology and Randomness

ABSTRACT: If you randomly drop points in space we seem to have trouble avoiding finding some pattern. This can cause mischief when we decide that some place is dangerous (there's a cancer cluster) or that there's a hole in a tissue because it seems to be not simply connected. I'll try to explain some interesting phenomena and introduce "topological data analysis"

Special talk (July 7): Trevor Hyde

TITLE: Arithmetic Topography

ABSTRACT: Many natural problems in number theory lead to the study of integral binary quadratic forms: polynomials of the form $Q(x, y) = ax^2 + bxy + cy^2$ for integers a, b, c. Crack open a classical book on the subject and prepare yourself for lots of thinly motivated calculations and manipulations. In this talk we use John Conway's brilliant topographical interpretation of integral binary quadratic forms to reach for the forest past the trees.

Logic (Week 4): Denis Hirschfeldt (Zoom, but in classroom)

TITLE: The metamathematical strength of Hindman's Theorem

ABSTRACT: Hindman's Theorem states that for any coloring of the natural numbers with finitely many colors, there is an infinite set S such that all sums of finitely many distinct elements of S have the same color. It has several proofs. Some, like Hindman's original proof, are rather complicated. Others, including one through ultrafilters, are simpler, but share with all other proofs of Hindman's Theorem the use of methods that are fairly "high-powered", in certain senses that can be formalized using ideas from mathematical logic. It is an open question whether such methods are necessary to prove Hindman's Theorem. We will discuss how to make this and related questions on the metamathematics of Hindman's Theorem precise from the points of view of computability theory and proof theory, in particular the program of reverse mathematics. In this way, we will present a case study in the computability-theoretic and reverse-mathematical study of mathematical principles that is particularly interesting in that it involves questions of current research interest.

Algebraic Topology: Peter May and others

Continuing: A guess is that we will introduce stable homotopy theory and maybe describe homotopical analogues of the monadicity theorem that are very important in algebraic topology. At some point I hope to explain how in principle the classifying spaces of symmetric groups "determine" the stable homotopy groups of spheres.

TITLE: Coambiguous concepts

ABSTRACT: We will start with some categorical linguistics, introducing the categorical language required for comparisons between subjects and comparisons between concepts. "Coambiguous concepts" are different definitions that non-obviously have the same or equivalent content. The idea leads to seriously interesting comparisons, either relating different areas of mathematics or giving alternative perspectives on a single area. The first week, we will introduce the categorical language that allows us to make such comparisons. Don't be put off by the undefined terms below!

Example: Finite T_0 topological spaces and finite posets are isomorphic concepts By contrast, what do we mean by equivalent concepts?

Example: Finite dimensional real vector spaces and their maps are equivalent to finite real matrices and matrix multiplication

Example: Finite sets are equivalent to the set of sets $\mathbf{n} = \{1, \cdots, n\}$

Example: Groups are equivalent to free groups modulo relations

Generalization next week: Monads and their algebras; the categorical "monadicity theorem".

TITLE: Homotopically coambiguous definitions

More deeply, what do we mean by concepts that are homotopically coambiguous, by which we mean that different definitions give homotopically equivalent concepts?

Example: Topological spaces, simplicial sets, small categories, and posets are homotopically equivalent concepts.

Example: Simplicial abelian groups and chain complexes are homotopically equivalent concepts.

Possible topics to be explored in more depth later on

Topic 1: Finite spaces and larger contexts (book in progress)

Topic 2: An introduction to stable homotopy theory and spectra

Slide talk: `https://www.youtube.com/watch?v=vRsrCNLkSA0`

Topic 3: An introduction to equivariant homotopy and cohomology theory

Slide talk: `https://www.cornell.edu/video/peter-may-equivariant-cohomology`

Topic 4: Operads and their algebras

Old new example: n-connective spaces are homotopically equivalent to "monadically coequalized" n-fold suspensions of E_n-spaces.

Old new example: Connective spectra are homotopically equivalent to "monadically coequalized" suspensions of E_∞-spaces.