AN INTRODUCTION TO NON-SYNCHRONOUS TRADING

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Abstract. The *non-synchronous trading effect* arises when time-series data asset prices are recorded at time intervals of irregular lengths. This paper first introduces Lo and MacKinlay's non-synchronous trading model and then computes moments of observed asset returns to evaluate the magnitude of non-synchronous trading effect.

Contents

1. Introduction

Different stocks have different trading frequencies. Therefore, the same market driven event can result in price changes of different stocks at various times. Yet, this time discrepancy is often ignored when analyzing stock prices. For example, daily prices quoted in financial databases are daily *closing* prices, i.e., last trading prices on a day. Since the time at which the last transaction occurs can vary from day to day, daily prices are not equally spaced at 24-hour intervals.

The *non-synchronous trading effect* occurs when prices are assumed to be recorded at fixed intervals when they are in fact recorded at intervals of varying lengths. This assumption is especially problematic when stocks have low trading frequencies. It can lead to erroneous inferences for stock returns. In particular, it results in misleading autocorrelation (i.e., correlation of the same stock at different time) and cross-correlation (i.e., correlation between different stocks) in stock returns.

To give a simple example of the non-synchronous trading effect, consider stock A and stock B whose returns are independent at each time period. Stock A trades actively, while stock B receives little market attention and rarely trades. Suppose breaking news hits the market and affects the aggregated stock market. As a result, stock A's and stock B's intrinsic values change. However, they may not be reflected in their market price, depending on whether individuals in the market choose to respond to the breaking news. If this news arrives near the end of a trading day, it is possible that only A is traded after the news and B is not. Therefore, only A's daily closing price shows this information and reflects A's intrinsic value. When

people eventually realize the change in stock B and trade B, stock B's market price will change accordingly to reflect its true price, albeit with a time lag.

This lagged response of B induces serial correlation between returns of stock A and stock B, though they are assumed to be independent temporally. Also, when not traded, stock B's observed daily return is zero. When traded, it reverts to the cumulative mean return, i.e., mean return of all prior consecutive periods in which it did not trade. This oscillation creates negative serial correlation in B's observed returns, which gives the false impression that we can predict B's future price to move in the opposite direction of its current price.

2. A Model of Non-synchronous Trading

In this section, we introduce Lo and MacKinlay's model to analyze the effects of non-synchronous trading from a statistical perspective.

Suppose trading is like a sequence of flipping coins. Consider a market where at each period t, the probability of not trading a security i is $\pi_{i,t}$, which is independent from all the other random variables, especially returns $r_{i,t}$.

Definition 2.1. Let $p_{i,t}$ denote the last price that a security i is traded at time interval t. The *observed market return* $r^o_{i,t}$ of i is

$$r^o_{i,t} = \log(p_{i,t}/p_{i,t-1}) \tag{2.1}$$

Since the market price might not reflect i's intrinsic value, it is possible that the observed market return differs from the virtual return (i.e., the real return computed with intrinsic values). The consistency between a security i's observed return $r^o_{i,t}$ and virtual return $r_{i,t}$ depends on whether i trades at period t. If not, the closing price $p_{i,t}$ is set to the previous period's closing price $p_{i,t-1}$, and its observed return $r^o_{i,t} = \log(p_{i,t}/p_{i,t-1}) = \log 1 = 0$. On the other hand, if i trades at interval t, its observed return is the sum of virtual returns at t and all prior consecutive periods in which it did not trade.

For example, consider a sequence of four consecutive periods. A security i only trades at period 1, 3, 4. The observed returns is equal to its virtual returns at period 1, 4, is equal to zero at period 2, and is equal to the sum of virtual returns of period 2 and 3 at period 3. To illustrate,

$$r^o_{i,t}(t) = \begin{cases} r_{i,1} & t = 1 \\ 0 & t = 2 \\ r_{i,2} + r_{i,3} & t = 3 \\ r_{i,4} & t = 4 \end{cases} \tag{2.2}$$

To continue with the model, consider a collection of N securities. Their virtual returns follow a one-factor linear model.

$$r_i, t = \mu_i + \beta_i f_t + \epsilon_{i,t} \qquad i = 1, ..., N \tag{2.3}$$

where f_t is the common factor with mean equal to zero (i.e., $E[f_t] = 0]$), and $\epsilon_{i,t}$ is the idiosyncratic error with mean equal to zero (i.e., for a security i, $E[\epsilon_{i,t}] = 0]$). It is is temporally and cross-sectionally independent, which means for security i, $\epsilon_{i,t}$ are independent of $\epsilon_{i,t-k}$ for all i, t, and $k \neq 0$. Also, for ease of analysis, we set the common factor f_t to be *i.i.d.* and independent of $\epsilon_{i,t-k}$ for all i, t, and k.

To directly work with the observed returns and to deduce non-synchronous trading's properties, we introduce the following two *Bernoulli* random variables.

Definition 2.2. Assume that $\delta_{i,t}$ is an *i.i.d.* random sequence for $i = 1, 2, ...N$. We can define $\delta_{i,t}$ and $X_{i,t}(k)$ as

$$\delta_{i,t} = \begin{cases} 1 & \text{(not traded)} \quad \text{with probability } \pi_i \\ 0 & \text{(traded)} \quad \text{with probability } 1 - \pi_i \end{cases} \tag{2.4}$$

$$X_{i,t}(0) \equiv 1 - \delta_{i,t} \tag{2.5}$$

Because $\delta_{i,t}$ is independent of $\delta_{j,t}$ for $i \neq j$, we then find $X_{i,t}(k)$

$$\begin{aligned} X_{i,t}(k) &\equiv (1 - \delta_{i,t})\delta_{i,t-1}\delta_{i,t-2}...\delta_{i,t-k}, \quad k > 0 \\ &= \begin{cases} 1 & \text{with probability } (1 - \pi_i)\pi_i^k \\ 0 & \text{with probability } 1 - (1 - \pi_i)\pi_i^k \end{cases} \end{aligned} \tag{2.6}$$

where $\delta_{i,t}$ is the indicator variable for whether asset i trades in period t: it is equal to 1 if i trades at t; otherwise, it is equal to 0. $X_{i,t}(k)$ is also an indicator variable. It is equal to 1 if asset i trades in period t but has not traded in-between previous consecutive periods $t - k$ and $t - 1$ and is 0 otherwise. For large enough k, $X_{i,t}(k)$ is almost certainly 0, because it is unlikely the case that asset i trades today but never before.

Definition 2.3. The number of past consecutive periods that asset i not traded, denoted by $\tilde{k_t}$, is expressed as:

$$\tilde{k_t} = \sum_{k=1}^{\infty} \prod_{j=1}^{k} \delta_{i,t-j} \tag{2.7}$$

We then derive the expectation and variance of $\tilde{k_t}$

$$E[\tilde{k_t}] = \frac{\pi_i}{1 - \pi_i} \tag{2.8}$$

$$Var[\tilde{k_t}] = \frac{\pi_i}{(1 - \pi_i)^2} \tag{2.9}$$

For example, if $\pi_i = \frac{1}{2}$, asset i is expected to go without trading for one period at a time.

Definition 2.4. Intuitively, the observed returns $r^o_{i,t}$ are given by

$$r^o_{i,t} = \sum_{k=0}^{\tilde{k_t}} r_{i,t-k} \qquad i = 1, 2, ..., N \tag{2.10}$$

Another way to view observed returns is

$$r^o_{i,t} = \begin{cases} 0 & \text{with probability } \pi_i \\ r_{i,t} & \text{with probability } (1 - \pi_i)^2 \\ r_{i,t} + r_{i,t-1} & \text{with probability } (1 - \pi_i)^2\pi_i \\ r_{i,t} + r_{i,t-1} + r_{i,t-2} & \text{with probability } (1 - \pi_i)^2\pi_i^2 \\ \vdots & \vdots \\ r_{i,t} + r_{i,t-1} + \ldots + r_{i,t-k} & \text{with probability } (1 - \pi_i)^2\pi_i^k \\ \vdots & \vdots \end{cases} \tag{2.11}$$

It is shown from above that $r^o_{i,t}$ correlates to $r^o_{i,t-k}$, because it contains previous virtual return $r_{i,t-k}$ with non-zero possibility $(1-\pi_i)^2\pi_i^k$.

The most useful way to express the observed return $r^o_{i,t}$ is in terms of the indicator variable $X_{i,t}(k)$.

$$r^o_{i,t} = \sum_{k=0}^{\infty} X_{i,t}(k) r_{i,t-k} \qquad i = 1, 2, ..., N \tag{2.12}$$

If asset i does not trade within period t, then $X_{i,t}(k) = 0$ for all k and the observed return $r^o_{i,t} = \sum_0^\infty 0 = 0$. If asset i trades within period t, then its observed return is equal to the sum of virtual return at t and previous consecutive periods in which i is not traded.

3. Implications for Asset Returns

To see how non-synchronous trading affects individual asset returns, we compute the moments and co-moments of $r^o_{i,t}$, which depend on the moments of $Xi, t(k)$. We first recall some useful definitions for time-series data.

Definition 3.1. Given a stochastic process $X = \{X_t\}$, the autocovariance function gives the covariance of the process with itself at pairs of time periods. It is given by

$$\begin{aligned} C_{XX}(t, t-k) &= \mathrm{Cov}\left(X_t, X_{t-k}\right) \\ &= \mathrm{E}\left[\left(X_t - \mu\right)\left(X_{t-k} - \mu\right)\right] \\ &= \mathrm{E}\left[X_t, X_{t-k}\right] - \mu^2 \end{aligned} \tag{3.1}$$

where t and $t-k$ are two different time periods and $\mu = \mathrm{E}\left[X\right]$

Definition 3.2. The autocorrelation function shows the magnitudes of correlation and inter-dependency of the process between pairs of time periods. Given a stochastic process $X = \{X_t\}$, the autocorrelation function is expressed as

$$\begin{aligned} \rho_{XX}(t, t-k) &= \frac{E\left[\left(X_t - \mu\right)\left(X_{t-k} - \mu\right)\right]}{\sqrt{E\left[\left(X_t - \mu\right)^2\right] E\left[\left(X_{t-k} - \mu\right)^2\right]}} \\ &= \frac{\mathrm{Cov}\left(X_t, X_{t-k}\right)}{\sigma_t \sigma_{t-k}} \end{aligned} \tag{3.2}$$

where t and $t-k$ are two time periods, $\sigma_t^2 = \mathrm{Var}(X_t)$, and $\sigma_{t-k}^2 = \mathrm{Var}(X_{t-k})$

Definition 3.3. Given two stochastic processes $\{X(t)\}$ and $\{Y(t)\}$. Then the cross-covariance function measures the maginitudes of correlation and dependency of one process with the other at pairs of time periods. The cross-covariance K_{XY} is given by

$$\begin{aligned} \mathrm{K}_{XY}(t, t-k) &= \mathrm{Cov}(X_t, Y_{t-k}) \\ &= \mathrm{E}\left[\left(X_t - \mu_X\right)\left(Y_{t-k} - \mu_Y\right)\right] \\ &= \mathrm{E}\left[X_t Y_{t-k}\right] - \mu_X \mu_Y \end{aligned} \tag{3.3}$$

where t and $t-k$ are two time periods in time, $\mu_X = \mathrm{E}\left[X\right]$, and $\mu - Y = \mathrm{E}\left[Y\right]$

Proposition 3.1. For the non-synchronous trading model defined in Section 2, the observed returns $\{r^o_{i,t}\}$ have constant first moment:

$$\mathrm{E}[r^o_{i,t}] = \mu_i \tag{3.4}$$

Proof. Recalling definition 2.4. and the definition of expectation for discrete random variables,

$$\begin{aligned} E[r^o_{i,t}] &= \sum_{k=0}^{\infty} E[X_{i,t}(k) r_{i,t-k}] \\ &= \sum_{k=0}^{\infty} E[X_{i,t}(k)] E[r_{i,t-k}] \\ &= \mu_i \sum_{k=0}^{\infty} (1-\pi_i)\pi_i^k \end{aligned} \tag{3.5}$$

Therefore,

$$E[r^o_{i,t}] = \mu_i \tag{3.6}$$

□

The above result implies that the mean of observed returns is not affected by the non-synchronous trading.

Proposition 3.2. For a security i, the observed returns $\{r^o_{i,t}\}$ have finite variance:

$$\mathrm{Var}[r^o_{i,t}] = \sigma_i^2 + \frac{2\pi_i}{1-\pi_i}\mu_i^2 \tag{3.7}$$

where $\sigma_i^2 \equiv \mathrm{Var}[r_{i,t}]$

Proof. We first derive $E[{r^o_{i,t}}^2]$.

(3.8)

$$\begin{aligned} E[{r^o_{i,t}}^2] &= E[\sum_{k=0}^{\infty} X_{i,t}(k) r_{i,t-k} \cdot \sum_{j=0}^{\infty} X_{it}(j) r_{i,t-j}] \\ &= \sum_{k=0}^{\infty} E[X^2_{i,t}(k) r^2_{i,t-k}] + 2\sum_{k<j}\sum_{j} E[X_{i,t}(k) X_{i,t}(j) r_{i,t-k} r_{i,t-j}] \\ &= (\mu_i^2 + \sigma_i^2)\sum_{k=0}^{\infty}(1-\pi_i)\pi_i^k + 2\sum_{k<j}\sum_{j} E[X_{i,t}(k)X_{i,t}(j)] \cdot E[r_{i,t-k} r_{i,t-j}] \\ &= \mu_i^2 + \sigma_i^2 + 2\sum_{k<j}\sum_{j}(1-\pi_i)\pi_i^j[\mu_i^2 + \sigma_i^2\theta(k-j)] \ , \ \theta(x) \equiv \begin{cases} 0 & x \neq 0 \\ 1 & x = 0 \end{cases} \\ &= \mu_i^2 + \sigma_i^2 + 2\sum_{k=0}^{\infty}\sum_{j=k+1}^{\infty}(1-\pi_i)\pi_i^j[\mu_i^2 + \sigma_i^2\theta(k-j)] \\ &= \mu_i^2 + \sigma_i^2 + 2(1-\pi_i)\sum_{k=0}^{\infty}(\pi_i^{k+1}\sum_{j=0}^{\infty}\mu_i^2\pi_i^j) \\ &= \mu_i^2 + \sigma_i^2 + 2\frac{\pi_i}{1-\pi_i}\mu_i^2 \end{aligned}$$

$$\mathrm{Var}[r^o_{i,t}] = E[r^o_{i,t}] - E^2[r^o_{i,t}] = \sigma_i^2 + 2\frac{\pi_i}{1-\pi_i}\mu_i^2 \tag{3.9}$$

□

Therefore, variance increases if the security has non-zero mean observed returns.

Proposition 3.3. For any two observed returns of individual asset i in sequence $r^o_{i,t}$, their covariance depends only on their relative position and not on their absolute position. The covariance is given by

$$\mathrm{Cov}[r^o_{i,t}, r^o_{i,t+n}] = -\pi_i^n \mu_i^2 \text{ for } n > 0 \tag{3.10}$$

Proof.

$$\begin{aligned}
E[r^o_{i,t} r^o_{i,t+n}] &= E[\sum_{k=0}^{\infty} X_{it}(k) r_{i,t-k} \cdot \sum_{j=0}^{\infty} X_{it+n}(j) r_{i,t+n-j}] \\
&= \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} E[X_{it}(k) X_{it+n}(j) r_{i,t-k} r_{i,t+n-j}] \\
&= \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} E[X_{it}(k) X_{it+n}(j)] \cdot E[r_{i,t-k} r_{i,t+n-j}] \\
&= \sum_{k=0}^{\infty}\sum_{j=0}^{n-1} (1-\pi_i)^2 \pi_i^{k+j} E[r_{i,t-k} r_{i,t+n-j}] \\
&= \sum_{k=0}^{\infty}\sum_{j=0}^{n-1} (1-\pi_i)^2 \pi_i^{k+j} \mu_i^2 \\
&= (1-\pi_i^n)\mu_i^2
\end{aligned} \tag{3.11}$$

The autocovariance hence is given by

$$\mathrm{Cov}[r^o_{i,t}, r^o_{i,t+n}] = E[r^o_{i,t} r^o_{i,t+n}] - E[r^o_{i,t}]E[r^o_{i,t+n}] = -\pi_i^n \mu_i^2 \tag{3.12}$$

□

Hence, we can deduce asset i's autocorrelation:

$$\begin{aligned}
\mathrm{Corr}\left[r^o_{i,t}, r^o_{i,t+n}\right] &= \frac{Cov[r^o_{i,t}, r^o_{i,t+n}]}{Var[r^o_{i,t}]} \\
&= \frac{-\mu_i^2 \pi_i^n}{\sigma_i^2 + \frac{2\pi_i}{1-\pi_i}\mu_i^2} \quad \text{for } n > 0
\end{aligned} \tag{3.13}$$

By (3.8), (3.9), non-synchronous trading induces spurious non-positive autocorrelation in asset i's observed returns, which differs from *i.i.d.* observed returns in normal settings. Also, autocorrelation decays geometrically, which implies that we are most likely to be influenced by non-synchronous trading in very short time frames.

The formula above also allows us to compute the maximal absolute autocorrelation in individual portfolio returns due to non-synchronous trading. Since the non-positive continuous function (3.9) of π_i is zero at $\pi_i = 0$ and approaches zero as π_i approaches 1, there must be at least one local minimum for some π_i in interval $[0, 1)$. The minimum first-order autocorrelation of sequence $r^o_{i,t}$ is as follows

$$\text{Min}_{\{\pi_i\}} \text{Corr}\left[r^o_{i,t}, r^o_{i,t+1}\right] = -\left(\frac{|\xi_i|}{1+\sqrt{2}\,|\xi_i|}\right)^2 \tag{3.14}$$

where $\xi_i \equiv \frac{\mu_i}{\sigma_i}$.

The autocorrelation reaches its minimum when

$$\pi_i = \frac{1}{1+\sqrt{2}\,|\xi_i|} \tag{3.15}$$

For all $\pi_i \in [0, 1)$ and $\xi_i \in (-\,\text{co}, +\infty)$, the minimum is therefore given as

$$\begin{aligned} \text{Inf}_{\{\pi_i,\xi,\}} \text{Corr}\left[r^o_{i,t}, r^o_{i,t+1}\right] &= \lim_{\xi_i \to \infty} -\left(\frac{|\xi_i|}{1+\sqrt{2}\,|\xi_i|}\right)^2 \\ &= -\frac{1}{2} \end{aligned} \tag{3.16}$$

Although correlation of $-\frac{1}{2}$ is statistically significant, it is virtually impossible for any empirical values. $-\frac{1}{2}$ is only attained when $|\xi_i|$ approaches positive infinity but never when $|\xi_i|$ is finite. A case resembling the reality is as follows.

Consider a period to be one trading day. Suppose μ_i is 0.05%, and σ_i is 2.5%. Then ξ_i is equal to 0.02, which is a typical value in reality. By(4.1), the autocorrelation in individual asset returns reaches its minimum -0.037% when probability of not trading is set to 97.2%, which implies an expected nontrading duration of 35.4 days.

Proposition 3.4. The cross-covariance between observed returns of asset i and asset j is expressed as

$$\text{Cov}[r^o_{i,t}, r^o_{j,t+n}] = \frac{(1-\pi_i)(1-\pi_j)}{1-\pi_i\pi_j}\beta_i\beta_j\sigma^2_\lambda\pi^n_j \text{ for } i \neq j, n \geq 0. \tag{3.17}$$

where $\sigma^2_\lambda = Var[f_t]$

Proof. Similar to (3.4), we first calculate $E\left[r^o_{i,t}r^o_{i,t+n}\right]$

$$E\left[r_{i,t-k}r_{i,t+n-l}\right] = \mu_i\mu_j + \beta_i\beta_j\sigma^2_\lambda\theta(l-k-n)\ ,\ \theta(x) \equiv \begin{cases} 0 & x \neq 0 \\ 1 & x = 0 \end{cases} \tag{3.18}$$

Then

(3.19)

$$
\begin{aligned}
E\left[r_{i,t}^{o} r_{i,t+n}^{o}\right] &= E\left[\sum_{k=0}^{\infty} X_{it}(k) r_{i,t-k} \cdot \sum_{l=0}^{\infty} X_{jt+n}(l) r_{i,t+n-l}\right] \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} E\left[X_{it}(k) X_{jt+n}(l) r_{i,t-k} r_{i,t+n-l}\right] \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} E\left[X_{it}(k)\right] \cdot E\left[X_{jt+n}(l)\right] \cdot E\left[r_{i,t-k} r_{i,t+n-l}\right] \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left(1-\pi_i\right) \pi_i^k \left(1-\pi_j\right) \pi_j^l \cdot \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left(1-\pi_i\right) \pi_i^k \left(1-\pi_j\right) \pi_j^l \mu_i \mu_j + \\
&\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left(1-\pi_i\right) \pi_i^k \left(1-\pi_j\right) \pi_j^l \beta_i \beta_j \sigma_\lambda^2 \theta \left(l - \beta_i \beta_j \sigma_\lambda^2 \theta (l-k-n)\right] \\
&= \mu_i \mu_j + \sum_{k=0}^{\infty} \left(1-\pi_i\right)\left(1-\pi_j\right) \beta_i \beta_j \sigma_\lambda^2 \pi_i^k \pi_j^{k+n} \\
&= \mu_i \mu_j + \left(1-\pi_i\right)\left(1-\pi_j\right) \beta_i \beta_j \sigma_\lambda^2 \pi_j^n \sum_{k=0}^{\infty} \left(\pi_i \pi_j\right)^k \\
&= \mu_i \mu_j + \frac{\left(1-\pi_i\right)\left(1-\pi_j\right)}{1-\pi_i \pi_j} \beta_i \beta_j \sigma_\lambda^2 \pi_j^n
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\operatorname{Cov}\left[r_{i,t}^{o}, r_{j,t+n}^{o}\right] &= E\left[r_{i,t}^{o} r_{j,t+n}^{o}\right] - E\left[r_{i,t}^{o}\right] E\left[r_{j,t+n}^{o}\right] \\
&= \frac{\left(1-\pi_i\right)\left(1-\pi_j\right)}{1-\pi_i \pi_j} \beta_i \beta_j \sigma_\lambda^2 \pi_j^n
\end{aligned} \tag{3.20}
$$

□

From (3.17), the sign of cross-covariance only depends on the signs of β_i and β_j, because all the other terms are positive. Also, because of the term π_j^n, this expression is asymmetric with respect to i and j. For example, suppose $\pi_i = 0$ and $\pi_j \neq 0$. Then cross-covariance is non-zero from only one side,

i.e., $$\begin{cases} \operatorname{Cov}\left[r_{i,t}^{o}, r_{j,t+n}^{o}\right] = (1-\pi_j)\beta_i \beta_j \sigma_\lambda^2 \pi_j^n \\ \operatorname{Cov}\left[r_{j,t}^{o}, r_{i,t+n}^{o}\right] = 0 \end{cases} \quad \text{for any } n > 0.$$

Recall the univariant linear model (2.3). When a security j does not trade, the returns to a frequently traded security i can be used to forecast j at $t+n$, because the common factor f_t presents in i's return and j's lagged return. On the other hand, the security j's future observed return does not include its current virtual return $r_{j,t}$ due to non-trading. Yet, security i's future observed return is related to its current virtual return and hence does not contain a common factor with j. The cross-covariance between $\{j,t\}$ and $\{i,t+n\}$ is zero.

Note that the only source of asymmetry in cross-covariance is the cross-sectional differences in the probabilities of non-synchronous trading. We then devise a way to calculate non-trading probabilities from cross-covariance.

We denote observed returns of N securities as a vector:

$$\boldsymbol{r}^o = [r^o_{1,t}, r^o_{2,t}, ..., r^o_{N-1,t}, r^o_{N,t}]' \tag{3.21}$$

The covariance matrix between N securities is then given by

$$\boldsymbol{\Gamma}_n = \mathrm{E}\left[\left(\mathbf{r}^o_t - \boldsymbol{\mu}\right)\left(\mathbf{r}^o_{t+n} - \boldsymbol{\mu}\right)'\right] \tag{3.22}$$

where $\boldsymbol{\mu} \equiv \mathrm{E}\left[\mathbf{r}^o_t\right]$

The (i, j)th element of matrix $\boldsymbol{\Gamma}_n$ is the cross-covariance function of two securities i and j and is given by

$$\gamma_{ij}(n) = \frac{(1-\pi_i)(1-\pi_j)}{1-\pi_i\pi_j}\beta_i\beta_j\sigma_f^2\pi_j^n \tag{3.23}$$

If the probabilities of non-trading are different between two securities i and j, the covariance matrix is asymmetric, and

$$\frac{\gamma_{ij}(n)}{\gamma_{ji}(n)} = \left(\frac{\pi_j}{\pi_i}\right)^n \tag{3.24}$$

When given the covariance matrix, to estimate nontrading probabilities, we only need one probability, say π_1, and we can estimate the rest using (3.24).

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