PROGRAM NOTES AND ABSTRACTS FOR WEEK 1

Program Notes

Each group (or "subprogram") will have a first general meeting sometime Monday or Tuesday.

Apprentice Program: Daniil Rudenko

We will cover a variety of topics in algebra, geometry and combinatorics. Each week there will be three (asynchronous) video lectures and three (synchronous) problem solving classes at times to be determined. More details will be given Monday morning.

Probability Subprogram: Gregory Lawler

There will be talks on various aspects of probability by a number of faculty throughout the REU on MWF 2:30, open to all.

On TT 2:30 the probability group will meet for more informal sessions, chances to ask questions, and so on. These are open to anyone, even those not in the probability group. For the first Tuesday, we will start by an informal session getting to know each other.

This week and next, Greg Lawler will speak on Harmonic functions, Brownian motion, and analysis in the plane. Abstract below. Note that these talks will also be of special interest to those focusing on analysis.

Algebraic Topology Subprogram: Peter May

There will be talks and more informal get togethers in algebraic topology and related areas throughout the REU. Some talks will be pitched towards a general audience, others will be more focused, but all will be open to all. The first Tuesday will start with an informal session. Suggestions of areas or topics that you would like to learn about are very welcome at any time, for example now. Last year's topics are recalled below. Participants from last year are invited to suggest less or more of any past topic.

Analysis, Dynamical Systems and Geometry Subprogram: Kasia Jankiewicz and Ao Sun

There will be talks and more informal get togethers in these areas throughout the REU. There will be a first get together of the analysis group and dynamical systems and geometry group Monday Tuesday morning. Gregory Lawler's first week talks should be viewed as part of the analysis subprogram, and more talks may be added, times TBD. Danny Calegari's and Howard Masur's first week talks should be viewed as part of the dynamical systems and geometry subprogram.

Algebraic Geometry, Number Theory, and Other Algebraic Subprogram: Trevor Hyde and Benedict Morrissey

There will be talks and more informal get togethers in these areas throughout the REU. There will be a first get together of the entire algebraic group Monday or Tuesday morning. A first algebra talk will be given Tuesday at 11:00 (time perhaps tentative).

Times and abstracts for the first week talks
All times are CST

MTWThF 1:00: Danny Calegari

Title: Introduction to complex dynamics

Abstract: Let $P(z) := z^d + a_1 z^{d-1} + a_2 + .. + a_d$ be a degree d polynomial in the variable z, where z and the a_j are all complex numbers. We can think of P as a map from the complex numbers $\mathbb{C}$ to itself, and we can study what happens when we iterate this map, that is, apply it over and over again. The study of the dynamics of iterated polynomials will lead us to many interesting and beautiful objects: Julia sets, the Mandelbrot set, Dynamical laminations, and so on. We will explain what these words mean, and talk about some of their properties. There will also be many pictures.

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MWF 2:30: Gregory Lawler

Title: Harmonic functions, Brownian motion, and analysis in the plane

I will consider analysis in the two-dimensional plane. We will start with the notion of a harmonic function: a function whose value at a point is equal to the "mean value" on a curve surrounding the point. Trying to understand mean value leads to Brownian motion (continuous random walk). We then use the special properties of two dimensions to define a conjugate harmonic function and the notion of a conformal transformation. We will show how to use Brownian motion to determine when two open sets in $\mathbb{R}^2$ are conformally invariant. I will assume knowledge of multivariable calculus but little more. At some point I will introduce "$\sqrt{-1}$" and mention how this theory is often phrased ("complex analysis") but there is nothing imaginary about the theory so I will downplay this aspect.

TTh 2:30: Probability group meetings

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Monday 4:00: Howard Mazur

Title: The mathematics of playing pool

Abstract: The study of the motion of a ball on a pool table leads to interesting mathematics. It is an example of what is called a dynamical system. I will look at a few of the interesting questions that arise and what one can say about them. This talk is aimed at a general audience.

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Tuesday 11:00: Trevor Hyde

Title: Combinatorial species

Abstract: Combinatorics, at its core, is about counting structures built from finite sets: trees, lists, partitions, permutations, graphs, etc. The theory of combinatorial species gives us a robust toolkit for counting such structures by way of generating functions. In this talk I will introduce the theory of species through many examples, culminating in Joyal's beautiful enumeration of the number of trees built from a set with n elements.

TTh 3:00: Maryanthe Malliaris

Title: Ultrafilters in mathematical logic

Abstract: Ultrafilters give a way of defining an infinite average of mathematical objects, via ultraproducts. We'll look at the basics of the definition and discuss how changing the ultrafilter may or may not change the average. In the second week, we'll look at the Keisler-Shelah isomorphism theorem, which uses ultraproducts to give a characterization of elementary equivalence from first-order logic. Little will be assumed and much will be defined. Time permitting, there will be pointers to related theorems in other areas (of Arrow, Ax-Grothendieck, Ramsey) and open questions.

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TTh 4:30, WF 4:00: Peter May (Introductory meeting Tuesday

Title: General audience talks on "coambiguous concepts"

The theme this summer will be explaining how "coambiguous concepts" — different definitions that non-obviously have the same or equivalent content — can lead to seriously interesting comparisons, either relating different areas of mathematics or giving alternative perspectives on a single area. The first week, we will give a sneaky introduction to categorical language that allows us to make such comparisons.

Example: Finite topological spaces (T_0) and finite posets are isomorphic concepts By contrast, what do we mean by equivalent concepts?

Example: Finite dimensional real vector spaces and their maps are equivalent to finite real matrices and matrix multiplication

Example: Finite sets are equivalent to the set of sets $\mathbf{n} = \{1, \cdots, n\}$

Example: Groups are equivalent to "connected groupoids"

Title: Homotopically coambiguous definitions

More deeply, what do we mean by concepts that are homotopically coambiguous, by which we mean that different definitions give homotopically equivalent concepts?

Example: The following concepts are homotopically equivalent: topological spaces, simplicial sets, small categories, and posets.

Example: Simplicial abelian groups and chain complexes are homotopically equivalent concepts.

Possible topics to be explored in more depth

Topic 1: Finite spaces and larger contexts (book in progress)

Topic 2: Classifying spaces and characteristic classes (book in progress)

Example: Bundles are equivalent to certain homotopy classes of maps

Topic 3: An introduction to stable homotopy theory and spectra

Slide talk: https://www.youtube.com/watch?v=vRsrCNLkSA0

Topic 4: An introduction to equivariant homotopy and cohomology theory

Slide talk: https://www.cornell.edu/video/peter-may-equivariant-cohomology

Topic 5: Operads, operad pairs, and their algebras

Old new example: n-connective spaces are homotopically equivalent to twisted n-fold suspensions of E_n-spaces.

Old new example: Connective spectra are homotopically equivalent to twisted suspensions of E_∞-spaces.

Topic 6: Bott periodicity

Slide talk: https://www.youtube.com/watch?v=NrZ61vKBdJA