The catenoid

- **Catenoid:**
  \[
  \left\{ (x, y, z) \mid x^2 + y^2 = \cosh^2 (z) \right\}
  \]
  (Euler 1744, Meusnier 1776).

- **Surface of revolution:** rotation of
  \[
  \left\{ (x, 0, z) \mid x = \cosh(z) \right\}
  \]
  around the z axis.

- **Unique non-planar minimal surface of revolution.**

Figure: A catenoid, by Matthias Weber, indiana.edu/~minimal/archive
Examples of minimal surfaces

The helicoid

- **Helicoid:**

  \[ \Sigma = \{(t \cos s, t \sin s, s) \mid s, t \in \mathbb{R}\} \]

  (Meusnier 1776).

- **Ruled surface:** \( \Sigma \cap \{z = s\} \) are lines.

- **Unique non-planar** ruled minimal surface (Catalan 1830).

*Figure: A helicoid, by Matthias Weber, indiana.edu/~minimal/archive*
Scherk’s surface

- **Scherk’s surface:**
  
  \[
  \{(x, y, z) \mid z = \log \frac{\cos y}{\cos x}\}
  \]

  (Scherk 1835).

- Graph defined over “checkerboard” in \(\mathbb{R}^2\).

- **Doubly-periodic** surface: invariant by translations.

Figure: Scherk’s surface, by Matthias Weber, indiana.edu/~minimal/archive
Scherk’s surface

- Related graph (in the hyperbolic plane $\mathbb{H}$) was used by H. Rosenberg and P. Collins, in 2007, to construct a harmonic diffeomorphism $f : \mathbb{C} \to \mathbb{H}$

Figure: Scherk’s surface, by Matthias Weber, indiana.edu/~minimal/archive
Scherk’s singly-periodic surface

- **Scherk’s singly-periodic surface:**

  \[ \{(x, y, z) \mid \sin z = (\sinh x)(\sinh y)\} \]

  (Scherk 1835).

- **Q:** How to make the union of two planes into a minimal surface?

- **Desingularization:** Important technique to construct examples (N. Kapouleas).

Figure: Scherk’s singly-periodic surface, by Matthias Weber, indiana.edu/~minimal/archive
Some remarks

- Many other examples on Matthias Weber’s page: indiana.edu/~minimal/archive.
- The investigation of minimal surfaces is closely related to the development of **Calculus of Variations** and **Geometric Measure Theory**.
- Even though minimal surfaces and hard to construct, recent progress shows that they are actually **abundant!** Variational techniques: Marques, Neves, Song, Liokumovich, Irie, Zhou.
- Some references: