ABSTRACTS FOR FIFTH WEEK PROGRAM, JULY 20–JULY 24

1:00 CENTRAL STANDARD TIME, Monday, Wednesday, and Friday

GREGORY LAWLER

PROBABILITY, RANDOM FIELDS, AND GEOMETRY IN STATISTICAL PHYSICS

ABSTRACT (from first week): We will be looking at models that arise in critical phenomena in statistical physics. The general framework is that there is a collection of sites and there is a random "field" defined on the sites. This field can be either a collection of random variables indexed by the sites or a random path or subgraph.

The lectures will focus on two main examples: the loop-erased walk which is closely related to uniform spanning trees and the Gaussian field. The lectures discuss the relationship of these to Markov chains and usual random walks, "loop measures". and determinants of the Laplacian. There are other models that participants may consider such as Ising model, percolation, and Potts models.

Other participants may consider the continuous analogues of these fields and random curves, and, in particular, the Schramm-Loewner evolution and the definition of the determinant of the Laplacian in the continuum. Other possibilities are random geometry (quantum gravity) and physics approaches to conformal field theory.

We will use facts in undergraduate mathematics from the following areas: linear algebra, (post-calculus) probability, real variables, complex variables, combinatorics and graph theory. These should not be considered strict prerequisites but they give hints to outside reading that participants may have to do.

The mathematics in the discrete models will involve a lot of combinatorics and should be of interest for those who like this kind of mathematics. The continuous analogues involve a lot of analysis (real, complex, and stochastic) and some PDE. It is not required to have much background in this; indeed, many find learning the discrete theory to be a good start before attacking the mathematically more sophisticated continuous theories.

It is hoped that the lecture notes for this class will become a book.

1:00 CENTRAL STANDARD TIME, Tuesday and Thursday

FOCUS GROUPS
Send email to host to request access

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PROBABILITY

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ALGEBRAIC TOPOLOGY

2:30 CENTRAL STANDARD TIME

TEENA GERHARDT, Monday, July 20

ALGEBRA IN TOPOLOGY AND TOPOLOGY IN ALGEBRA
Title: Algebra in topology and topology in algebra Abstract:

ABSTRACT: How do we quantify the difference between the surface of a basketball and the surface of a doughnut? Algebraic topology is a branch of mathematics that uses the tools of algebra to study and distinguish topological spaces. But the tools of topology can also be used to study objects in algebra. In this talk we will explore the fascinating interplay between algebra and topology, and see how it is manifested in a tool called algebraic K-theory.

MARK BEHRENS Wednesday July 22 and Friday July 24, 2:30

TALK 1: K-THEORY

ABSTRACT: I will introduce K-theory, a cohomology theory based on vector bundles.

TALK 2: The Hopf invariant 1 problem

ABSTRACT: I will discuss the Hopf invariant 1 problem, and explain an "easy" solution of this problem by Adams and Atiyah using K-theory.

Zihui Zhao, Tuesday and Thursday July 21 and 23, 2:30

HARMONIC MEASURE

ABSTRACT: Given a domain Ω, its harmonic measure is a probability on the boundary and it measures where an object following Brownian motion exits from Ω. It also allows us to write down solutions to boundary value problems of the Laplacian equation. In this mini-course, I will show how we can look at harmonic measures from the perspectives of probability theory, complex analysis and elliptic PDEs. I will also introduce notions that characterize the regularity and dimension of a *measure*, and illustrate these notions using the example of the harmonic measure (in particular as compared to the canonical surface measure on $\partial\Omega$).

4:00 CENTRAL STANDARD TIME

MIKE HILL, Tuesday July 21, 4:00

NORMS IN EQUIVARIANT ALGEBRA AND HOMOTOPY THEORY

ABSTRACT: If M is a G-module, then the various fixed points of M naturally assemble into a Mackey functor: the restriction maps are the natural inclusions while the transfers are given by summing up over cosets. If M is actually a commutative ring on which G acts by commutative ring maps, then we have a multiplicative version of this transfer, where we multiply over the cosets instead. Evens defined a generalization of this to group cohomology (the derived functors of fixed points), and Tambara found a general framework to describe Mackey functors together with a compatible family of multiplicative transfers. In this talk, I'll introduce the concept of a Tambara functor, describing some of the basic examples. Time permitting, I will describe how this fits into the broader context of equivariant homotopy theory.

Thursday, July 23, 4:00

AGNES BEAUDRY

The return of ANTS ON PANTS (by popular demand)

ABSTRACT: (From Week 1) In this talk, I will give an introduction to manifolds and cobordism. What are manifolds? An ant living on a very large circle wouldn't know that it isn't living on the (flat) real line. Similarly, a d-manifold is a geometric object which, from an ant's perspective, looks flat like Euclidean space $\mathbb{R}^d$, but which, from a bird's-eye view, can look curved or otherwise interesting, like the unit sphere in $\mathbb{R}^{d+1}$.

What is cobordism? Think of a 2-dimensional surface that looks like a pair of empty pants. If the waist is the large circle which is the ant's universe, then the pants represent a transformation of the ant's world into a two circle universe. Similarly, a cobordism is a $d+1$ manifold with boundary which transforms one d-manifold into another. Two manifolds are cobordism equivalent if such a transformation exists.

An interesting and difficult question is that of classifying manifolds. A raw classification in arbitrary dimension is nearly impossible, and for this reason, mathematicians often settle for less precise answers. For example, can one classify manifolds up to cobordism equivalence? Come to my talk and find some answers to the ants on pants conundrum.

The talk will complete the picture begun the first week.

PETER MAY; Monday, Wednesday, and Friday, July 20, 22, 24

(see also Tuesday, Thursday 1:00 focus groups)

TOPICS IN AND AROUND ALGEBRAIC TOPOLOGY

For those new to algebraic topology here is an historically oriented introductory talk YouTube

ABSTRACTS (From first week; topics randomly dispersed in talks)

Topic 1: FINITE SPACES AND LARGER CONTEXTS

It is a striking, if esoteric, fact that in principle one can do all of algebraic topology using Alexandroff spaces, in which arbitrary intersections of open sets are open. That is even true equivariantly, when one considers spaces with symmetries given by group actions. Alexandroff spaces in which the topology distinguishes points are the "same thing" as partially ordered sets (posets). This establishes a close connection between algebraic topology and combinatorics. There are many applications, some speculative, others far out. For example, finite spaces have been used to study the role of RNA in evolution. I am writing a book on this subject Finite spaces and I am presenting topics from it and from past REU papers (maybe some of you might want to contribute).

Topic 2: CLASSIFYING SPACES AND CHARACTERISTIC CLASSES

This is classical algebraic topology that everyone should know but is seldom taught. The passage from geometric topology to algebraic topology passes through the equivalence of vector bundles with homotopy classes of maps into classifying spaces. Then characteristic classes are invariants of bundles that are determined by the cohomology of classifying spaces. This is also the subject of a book in progress Characteristic classes

Topic 3: EQUIVARIANT HOMOTOPY AND COHOMOLOGY THEORY

What is equivariant algebraic topology? In particular, what are equivariant cohomology theories and what are they good for? This is the subject of the book Equivariant homotopy and cohomology theory and of the talk Equiv

Topic 4: OPERADS, OPERAD PAIRS, AND THEIR ALGEBRAS

Modern abstract homotopy theory largely evolved from early work on iterated loop space theory: A_∞-spaces, E_n-spaces, and E_∞-spaces, and the spectra associarted to the last. This is a more advanced topic, but I may give it a try, focusing on the relationships among spaces, categories and "stable spaces" or spectra. Expository papers include What?, How?, and Why?.

Topic 5: SPECTRAL SEQUENCES