ABSTRACTS FOR FOURTH WEEK PROGRAM, JULY 13–JULY 17

1:00 CENTRAL STANDARD TIME, Monday, Wednesday, and Friday

GREGORY LAWLER

PROBABILITY, RANDOM FIELDS, AND GEOMETRY IN STATISTICAL PHYSICS

ABSTRACT (from first week): We will be looking at models that arise in critical phenomena in statistical physics. The general framework is that there is a collection of sites and there is a random "field" defined on the sites. This field can be either a collection of random variables indexed by the sites or a random path or subgraph.

The lectures will focus on two main examples: the loop-erased walk which is closely related to uniform spanning trees and the Gaussian field. The lectures discuss the relationship of these to Markov chains and usual random walks, "loop measures". and determinants of the Laplacian. There are other models that participants may consider such as Ising model, percolation, and Potts models.

Other participants may consider the continuous analogues of these fields and random curves, and, in particular, the Schramm-Loewner evolution and the definition of the determinant of the Laplacian in the continuum. Other possibilities are random geometry (quantum gravity) and physics approaches to conformal field theory.

We will use facts in undergraduate mathematics from the following areas: linear algebra, (post-calculus) probability, real variables, complex variables, combinatorics and graph theory. These should not be considered strict prerequisites but they give hints to outside reading that participants may have to do.

The mathematics in the discrete models will involve a lot of combinatorics and should be of interest for those who like this kind of mathematics. The continuous analogues involve a lot of analysis (real, complex, and stochastic) and some PDE. It is not required to have much background in this; indeed, many find learning the discrete theory to be a good start before attacking the mathematically more sophisticated continuous theories.

It is hoped that the lecture notes for this class will become a book.

1:00 CENTRAL STANDARD TIME, Tuesday and Thursday

FOCUS GROUPS
Send email to host to request access

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PROBABILITY

PETER MAY; may@math.uchicago.edu

ALGEBRAIC TOPOLOGY

10:00 and 2:30 CENTRAL STANDARD TIME

ASAF KATZ, Monday and Thursday, July 13 and July 16, 2:30

DYNAMICS AND NUMBER THEORY – A REMARKABLE CONNECTION

ABSTRACT: In recent years, many of the recent advances in number theory were achieved using ideas based on dynamical systems and ergodic theory. In these lectures I will survey the relationship between various questions in number theory such as the Oppenheim conjecture and questions regarding diophantine approximation, to questions in dynamics such as flows on homogeneous spaces and measure classification, following the works of Gregory Margulis and Marina Ratner. If time permits I will describe in rough lines some proofs around measure classification.

PEDRO GASPAR MARQUES DA SILVA and HENRIK MATTHIESEN, Tuesday July 14, 2:30 (last of 4 talks)

INTRODUCTION TO MINIMAL SURFACES IN $\mathbb{R}^3$ AND THE BERNSTEIN THEOREM

ABSTRACT: Minimal surfaces are among the most relevant objects in differential geometry and geometric analysis, with far-reaching connections to partial differential equations, topology, and general relativity. The lectures will cover classical aspects of minimal surfaces in Euclidean 3-space, aiming to prove Bernstein's Theorem on entire minimal graphs.

Lecture 4: Stability inequality and Bernstein's Theorem.

We assume some familiarity with multivariable calculus, including Gauss' (or divergence) Theorem. Basic knowledge about differential forms and Stokes' Theorem might be helpful, but is not strictly necessary. Depending on the participants' interest, we can later move to other topics (monotonicity formula, curvature estimates, maximum principle, stability and Morse index,...), possibly as a focus group.

REDIET ABEBE Wednesday, July 15, 10:00 am (Note the morning time)

MODELING THE DYNAMICS OF POVERTY

ABSTRACT: Poverty and economic hardship are understood to be highly complex and dynamic phenomena. Due to the multi-faceted nature of economic welfare, assistance programs targeted at alleviating hardship can face challenges, as they often rely on simpler measures of welfare, such as income or wealth, that fail to capture the full complexity of families? states. Here, we explore one important dimension ? susceptibility to income shocks. We introduce a model of welfare that incorporates income, wealth, and income shocks. We analyze this model to show that it can vary, at times substantially, from measures of welfare that only use income or wealth. We then study the algorithmic problem of optimally allocating subsidies in the presence of income shocks. We consider two well-studied objectives: the first aims to minimize the expected number of agents that fall below a given welfare threshold (a min-sum objective) and the second aims to minimize the likelihood that the most vulnerable agent falls below this threshold (a min-max objective). We present optimal and near-optimal algorithms for various general settings. We close with a discussion on future directions on allocating societal resources and the mechanism design for social good. See http://md4sg.com for the (MD4SG) research interface.

KATE PONTO Wednesday, July 15, 2:30

Strings and traces

ABSTRACT: An important part of doing math well is having good ways to explain it. I'm going to talk about string diagrams - an effective and fun way to communicate some pretty abstract ideas. I'll start by giving you the rules of the game; we will prove a few things with them and then (time permitting) I'll give you some idea of what we showed!

Friday, July 17, 2:30

BERTRAND GUILLOU

STEENROD OPERATIONS

ABSTRACT: Algebraic topologists like to study topological spaces by converting them into algebraic objects. In practice, the algebraic objects are often more approachable. One such process, known as cohomology, starts from a space and produces a (graded) commutative ring. It turns out, however, that these rings have an even richer structure: cohomology operations act on them as well.

I will discuss these cohomology operations, also known as Steenrod operations. Taken as a collection, they form the "Steenrod algebra", which is a very interesting algebraic object in its own right. We will draw pictures that give helpful ways to visualize Steenrod operations and will discuss some open problems.

4:00 CENTRAL STANDARD TIME

SHMUEL WEINBERGER, Tuesday and Thursday, July 14 and 16, 4:00

INTRODUCTION TO QUANTITATIVE TOPOLOGY

ABSTRACT: Algebraic topology tells us a lot about the existence of highly nonlinear maps between spaces. Quantitative topology tries to understand what these maps look like.

PETER MAY; Monday, Wednesday, and Friday, July 13, 15, 17

(see also Tuesday, Thursday 1:00 focus groups)

TOPICS IN AND AROUND ALGEBRAIC TOPOLOGY

For those new to algebraic topology here is an historically oriented introductory talk YouTube

ABSTRACTS (From first week; topics randomly dispersed in talks)

Topic 1: FINITE SPACES AND LARGER CONTEXTS

It is a striking, if esoteric, fact that in principle one can do all of algebraic topology using Alexandroff spaces, in which arbitrary intersections of open sets are open. That is even true equivariantly, when one considers spaces with symmetries given by group actions. Alexandroff spaces in which the topology distinguishes points are the "same thing" as partially ordered sets (posets). This establishes a close connection between algebraic topology and combinatorics. There are many applications, some speculative, others far out. For example, finite spaces have been used to study the role of RNA in evolution. I am writing a book on this subject Finite spaces and I am presenting topics from it and from past REU papers (maybe some of you might want to contribute).

Topic 2: CLASSIFYING SPACES AND CHARACTERISTIC CLASSES

This is classical algebraic topology that everyone should know but is seldom taught. The passage from geometric topology to algebraic topology passes through the equivalence of vector bundles with homotopy classes of maps into classifying spaces. Then characteristic classes are invariants of bundles that are determined by the cohomology of classifying spaces. This is also the subject of a book in progress Characteristic classes

Topic 3: EQUIVARIANT HOMOTOPY AND COHOMOLOGY THEORY

What is equivariant algebraic topology? In particular, what are equivariant cohomology theories and what are they good for? This is the subject of the book Equivariant homotopy and cohomology theory and of the talk Equiv

Topic 4: OPERADS, OPERAD PAIRS, AND THEIR ALGEBRAS

Modern abstract homotopy theory largely evolved from early work on iterated loop space theory: A_∞-spaces, E_n-spaces, and E_∞-spaces, and the spectra associarted to the last. This is a more advanced topic, but I may give it a try, focusing on the relationships among spaces, categories and "stable spaces" or spectra. Expository papers include What?, How?, and Why?.