ABSTRACTS FOR THIRD WEEK PROGRAM, JULY 6–JULY 10

1:00 CENTRAL STANDARD TIME, Monday, Wednesday, and Friday

GREGORY LAWLER

PROBABILITY, RANDOM FIELDS, AND GEOMETRY IN STATISTICAL PHYSICS

ABSTRACT (from first week): We will be looking at models that arise in critical phenomena in statistical physics. The general framework is that there is a collection of sites and there is a random "field" defined on the sites. This field can be either a collection of random variables indexed by the sites or a random path or subgraph.

The lectures will focus on two main examples: the loop-erased walk which is closely related to uniform spanning trees and the Gaussian field. The lectures discuss the relationship of these to Markov chains and usual random walks, "loop measures". and determinants of the Laplacian. There are other models that participants may consider such as Ising model, percolation, and Potts models.

Other participants may consider the continuous analogues of these fields and random curves, and, in particular, the Schramm-Loewner evolution and the definition of the determinant of the Laplacian in the continuum. Other possibilities are random geometry (quantum gravity) and physics approaches to conformal field theory.

We will use facts in undergraduate mathematics from the following areas: linear algebra, (post-calculus) probability, real variables, complex variables, combinatorics and graph theory. These should not be considered strict prerequisites but they give hints to outside reading that participants may have to do.

The mathematics in the discrete models will involve a lot of combinatorics and should be of interest for those who like this kind of mathematics. The continuous analogues involve a lot of analysis (real, complex, and stochastic) and some PDE. It is not required to have much background in this; indeed, many find learning the discrete theory to be a good start before attacking the mathematically more sophisticated continuous theories.

It is hoped that the lecture notes for this class will become a book.

1:00 CENTRAL STANDARD TIME, Tuesday and Thursday

FOCUS GROUPS
Send email to host to request access

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PROBABILITY

PETER MAY; may@math.uchicago.edu

ALGEBRAIC TOPOLOGY

Series of four talks:
Tuesday, Wednesday, and Friday July 7, 8, and 10 and Tuesday July 14, all at 2:30 CST

PEDRO GASPAR MARQUES DA SILVA and HENRIK MATTHIESEN
Pedro Gaspar Marques Da Silva

INTRODUCTION TO MINIMAL SURFACES IN $\mathbb{R}^3$ AND THE BERNSTEIN THEOREM

ABSTRACT: Minimal surfaces are among the most relevant objects in differential geometry and geometric analysis, with far-reaching connections to partial differential equations, topology, and general relativity. The lectures will cover classical aspects of minimal surfaces in Euclidean 3-space, aiming to prove Bernstein's Theorem on entire minimal graphs.

Lecture 1 (07/07): Area, minimal graphs, calibrations, and minimizing property.

Lecture 2 (07/10): Geometry of surfaces in $\mathbb{R}^3$, examples of minimal surfaces and first variation of the area

Lecture 3: First variation of the area (continued), harmonicity of coordinate functions; logarithmic cut-off trick.

Lecture 4: Stability inequality and Bernstein's Theorem.

We assume some familiarity with multivariable calculus, including Gauss' (or divergence) Theorem. Basic knowledge about differential forms and Stokes' Theorem might be helpful, but is not strictly necessary. Depending on the participants' interest, we can later move to other topics (monotonicity formula, curvature estimates, maximum principle, stability and Morse index,...), possibly as a focus group.

Tuesday and Thursday July 7 and 9, 4:00 CENTRAL STANDARD TIME

DANNY CALEGARI

GRAPHS AND GROUPS

ABSTRACT: We explain how an algebraic problem (understanding subgroups of free groups) is equivalent to a geometric/combinatorial problem, involving decorated graphs.

4:00 CENTRAL STANDARD TIME

PETER MAY; Monday, Wednesday, and Friday (also Tuesday, Thursday 1:00 focus groups)
individual or small group meetings by request or invitation

TOPICS IN AND AROUND ALGEBRAIC TOPOLOGY

For those new to algebraic topology here is an historically oriented introductory talk YouTube

Monday, July 6: AN INTRODUCTION TO THE HOMOLOGY OF SIMPLICIAL COMPLEXES, SIMPLICIAL SETS, TOPOLOGICAL SPACES, GROUPS, AND THEIR HISTORICAL DEVELOPMENT

ABSTRACTS (From first week; topics randomly dispersed in talks)

Topic 1: FINITE SPACES AND LARGER CONTEXTS

It is a striking, if esoteric, fact that in principle one can do all of algebraic topology using Alexandroff spaces, in which arbitrary intersections of open sets are open. That is even true equivariantly, when one considers spaces with symmetries given by group actions. Alexandroff spaces in which the topology distinguishes points are the "same thing" as partially ordered sets (posets). This establishes a close connection between algebraic topology and combinatorics. There are many applications, some speculative, others far out. For example, finite spaces have been used to study the role of RNA in evolution. I am writing a book on this subject Finite spaces and I am presenting topics from it and from past REU papers (maybe some of you might want to contribute).

Topic 2: CLASSIFYING SPACES AND CHARACTERISTIC CLASSES

This is classical algebraic topology that everyone should know but is seldom taught. The passage from geometric topology to algebraic topology passes through the equivalence of vector bundles with homotopy classes of maps into classifying spaces. Then characteristic classes are invariants of bundles that are determined by the cohomology of classifying spaces. This is also the subject of a book in progress Characteristic classes

Topic 3: EQUIVARIANT HOMOTOPY AND COHOMOLOGY THEORY

What is equivariant algebraic topology? In particular, what are equivariant cohomology theories and what are they good for? This is the subject of the book Equivariant homotopy and cohomology theory and of the talk Equiv

Topic 4: OPERADS, OPERAD PAIRS, AND THEIR ALGEBRAS

Modern abstract homotopy theory largely evolved from early work on iterated loop space theory: A_∞-spaces, E_n-spaces, and E_∞-spaces, and the spectra associarted to the last. This is a more advanced topic, but I may give it a try, focusing on the relationships among spaces, categories and "stable spaces" or spectra. Expository papers include What?, How?, and Why?.