ABSTRACTS FOR FIRST WEEK PROGRAM, JUNE 22–JUNE 26

1:00 CENTRAL STANDARD TIME, Monday, Wednesday, and Friday

GREGORY LAWLER

PROBABILITY, RANDOM FIELDS, AND GEOMETRY IN STATISTICAL PHYSICS

ABSTRACT: We will be looking at models that arise in critical phenomena in statistical physics. The general framework is that there is a collection of sites and there is a random "field" defined on the sites. This field can be either a collection of random variables indexed by the sites or a random path or subgraph.

The lectures will focus on two main examples: the loop-erased walk which is closely related to uniform spanning trees and the Gaussian field. The lectures discuss the relationship of these to Markov chains and usual random walks, "loop measures". and determinants of the Laplacian. There are other models that participants may consider such as Ising model, percolation, and Potts models.

Other participants may consider the continuous analogues of these fields and random curves, and, in particular, the Schramm-Loewner evolution and the definition of the determinant of the Laplacian in the continuum. Other possibilities are random geometry (quantum gravity) and physics approaches to conformal field theory.

We will use facts in undergraduate mathematics from the following areas: linear algebra, (post-calculus) probability, real variables, complex variables, combinatorics and graph theory. These should not be considered strict prerequisites but they give hints to outside reading that participants may have to do.

The mathematics in the discrete models will involve a lot of combinatorics and should be of interest for those who like this kind of mathematics. The continuous analogues involve a lot of analysis (real, complex, and stochastic) and some PDE. It is not required to have much background in this; indeed, many find learning the discrete theory to be a good start before attacking the mathematically more sophisticated continuous theories.

It is hoped that the lecture notes for this class will become a book.

1:00 CENTRAL STANDARD TIME, Tuesday and Thursday

FOCUS GROUPS

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CAROLINE TERRY; caterry@uchicago.edu

MODEL THEORY

ABSTRACT: Keisler's 1970 ICM paper Model Theory gives a short personal overview of the subject in its early days. The plan is to hold two group discussions during the first week to discuss the paper, how the field would grow from there, and all that remains to be said. The hope is that this will be useful to experienced and beginning logicians alike. Participants should be willing to look at the paper in advance and come prepared to discuss.

PETER MAY; may@math.uchicago.edu

ALGEBRAIC TOPOLOGY

First meeting of those who have expressed a strong interest in algebraic topology. Others are welcome.

2:30 CENTRAL STANDARD TIME

HOWARD MASUR, Monday

THE MATHEMATICS OF PLAYING POOL

ABSTRACT: The talk will be a gentle introduction to a number of mathematical ideas that arise in the study of billiards.

AGNES BEAUDRY, Tuesday

ANTS ON PANTS

ABSTRACT: In this talk, I will give an introduction to manifolds and cobordism.

What are manifolds? An ant living on a very large circle wouldn't know that it isn't living on the (flat) real line. Similarly, a d-manifold is a geometric object which, from an ant's perspective, looks flat like Euclidean space $\mathbb{R}^d$, but which, from a bird's-eye view, can look curved or otherwise interesting, like the unit sphere in $\mathbb{R}^{d+1}$.

What is cobordism? Think of a 2-dimensional surface that looks like a pair of empty pants. If the waist is the large circle which is the ant's universe, then the pants represent a transformation of the ant's world into a two circle universe. Similarly, a cobordism is a $(d + 1)$-manifold with boundary which transforms one d-manifold into another. Two manifolds are cobordism equivalent if such a transformation exists.

An interesting and difficult question is that of classifying manifolds. A raw classification in arbitrary dimension is nearly impossible, and for this reason, mathematicians often settle for less precise answers. For example, can one classify manifolds up to cobordism equivalence? Come to my talk and find some answers to the ants on pants conundrum.

DOUG RAVENEL, Wednesday

THE COFFEE CUP AND BELT TRICKS, AND THEIR HOMOTOPY THEORETIC EXPLANATIONS

ABSTRACT: The coffee cup trick is an illustration of how it is possible to give a full coffee cup two full revolutions without loosening your grip on it, without spilling your coffee, and (most importantly) without breaking your arm. In the belt trick we will see that giving a belt two full twists is in a certain sense equivalent to giving it no twists.

In both cases the mathematical explanation has to do with the topology of the special orthogonal groups (which I will define) in dimensions two and three. The key definition here is that of a homotopy or continuous deformation between two continuous maps $X \to Y$. It is one of the fundamental concepts of algebraic topology.

DISCLAIMER: No mathematicians were harmed during the production of this lecture. You can safely try it at home.

2:30 CENTRAL STANDARD TIME

EMILY RIEHL, Thursday

CATEGORIFYING CARDINAL ARITHMETIC

ABSTRACT: In this talk we'll prove that $a \times (b + c) = a \times b + a \times c$ via a roundabout method that takes us on a tour through several deep ideas including categorification, the Yoneda lemma, universal properties, and adjunctions.

(Prerequisites are just a bit of set theory, so apprentices are welcome.)

MONA MERLING, Friday

SOCIAL CHOICE AND TOPOLOGY

ABSTRACT: To avoid misleading anyone, this talk will not be about the sociology of topologists! "Social choice" is a model for decision making in economic, social, political contexts. For example: suppose that each person gets to vote on their favorite location where they would like to place a statue on an island. Is there a fair way based on these votes to choose the location? This will turn out to be a topological, even a homotopical problem, depending on the topology of the island. In this talk we will explore social choice models and fully answer the question about when they exist using algebraic topology.

The talk will serve as an advertisement for algebraic topology and basic category theory.

4:00 CENTRAL STANDARD TIME

AKHIL MATHEW, Tuesday and Thursday

QUADRATIC FORMS

ABSTRACT: I will discuss some aspects of the arithmetic of quadratic forms, including their classification over finite, local, and global fields, and the connection to more recent developments.

PETER MAY; Monday, Wednesday, and Friday

A NOT QUITE DETERMINED CHOICE OF TOPICS IN AND AROUND ALGEBRAIC TOPOLOGY

WARNING: Caveat emptor, ambivalent topics ahead
For those new to algebraic topology and with a strong stomach, somebody (not me) posted an historically oriented introductory talk YouTube

Topic 1: FINITE SPACES AND LARGER CONTEXTS
It is a striking, if esoteric, fact that in principle one can do all of algebraic topology using Alexandroff spaces, in which arbitrary intersections of open sets are open. That is even true equivariantly, when one considers spaces with symmetries given by group actions. Alexandroff spaces in which the topology distinguishes points are the "same thing" as partially ordered sets (posets). This establishes a close connection between algebraic topology and combinatorics. There are many applications, some speculative, others far out. For example, finite spaces have been used to study the role of RNA in evolution. I am writing a book on this subject Finite spaces and I may present topics from it and from past REU papers (maybe some of you might want to contribute).

Topic 2: CLASSIFYING SPACES AND CHARACTERISTIC CLASSES
This is classical algebraic topology that everyone should know but is seldom taught. The passage from geometric topology to algebraic topology passes through the equivalence of vector bundles with homotopy classes of maps into classifying spaces. Then characteristic classes are invariants of bundles that are determined by the cohomology of classifying spaces. This is also the subject of a book in progress.

Topic 3: EQUIVARIANT HOMOTOPY AND COHOMOLOGY THEORY
What is equivariant algebraic topology? In particular, what are equivariant cohomology theories and what are they good for? This is the subject of the book Equivariant homotopy and cohomology theory and of the talk Equiv

Topic 4: OPERADS, OPERAD PAIRS, AND THEIR ALGEBRAS
Modern abstract homotopy theory largely evolved from early work on iterated loop space theory: A_∞-spaces, E_n-spaces, and E_∞-spaces, and the spectra associarted to the last. This is a more advanced topic, but I may give it a try, focusing on the relationships among spaces, categories and "stable spaces" or spectra. Expository papers include What?, How?, and Why?.