NOTES ON THE 2016 REU PROGRAM

Here are some notes about the program. They will be updated throughout the program as more information comes in. The program is frontloaded, with the number of talks decreasing as the focus switches to your work on papers.

APPRENTICE PROGRAM: László Babai, Madhur Tulsiani, and Angela Wu

Weeks 1-5. I do not have an abstract for the complete 2016 Apprentice program, but let me refer you to Professor Babai's web page about past years give an idea.

http://people.cs.uchicago.edu/ laci/reu

Babai and Tulsiani are away week 1, and Angela Wu will fill in for them. She will be getting things started with a

"Mini-course in graph theory"

We introduce central ideas of graph theory, solving stand-alone problems while building intuition for the rest of the Apprentice program. The theme is the discovery through graph theory of deep connections between seemingly unrelated concepts and finding elegant proofs for surprising theorems.

To repeat from a previous email, the apprentice course will be more structured than in the past. In order to allow it to reach serious conceptual foundations and interesting applications in quasi-polynomial time (google "quasipolynomial time"), all apprentice participants will be expected to genuinely participate by turning in problem sets at prescribed times, and they will be given feedback on them. REU participants in the full program are also welcome to participate; there will be ample material in the course they will probably not have seen before.

Group work will be encouraged on more complex problems, while students will be expected to solve rudimentary problems individually. Apprentice papers are strongly encouraged to relate to the apprentice course. Professor Babai will prepare guidelines and proposed topics for these papers. One option will be to develop solutions to challenge problems or problem sequences into a coherent exposition. As stated in the announcement of the program, "apprentice papers not related to the lectures must be approved by the program director."

FULL PROGRAM: There will be sequences of talks in many areas. Some will last two or three week, others longer, and some changes from the following are likely, although the first two weeks are pretty much set.

PROBABILITY:

Week one: Gregory Lawler

"What is Stochastic Calculus?"

One of the most important ideas in elementary calculus is that we can compute a function f(t) knowing the current value and its rate of change. Stochastic calculus, which is used in physical and biological sciences as well as in modern mathematical finance, generalizes this by allowing for randomness in the evolution.

Week two: Antonio Auffinger

"From the Riemann hypothesis to self-driving cars."

I will discuss a few statistical physical models and their complexity. Remarkably, these models allow us to dig into one of the most famous problems in mathematics while explaining some properties of artificial intelligence.

Familiarity with probability and PDEs will be useful but is not essential.

ANALYSIS: Stanley Snelson and Max Engelstein

Weeks one and two (at least)

"Introduction to Calculus of Variations"

Calculus of variations, the science of minimizing functionals, began in the 17th and 18th centuries with the work of Euler and the Bernoullis, and is still an active area of research today. Minimizing a functional (such as the length of a curve or the kinetic energy of a vibrating membrane) can be thought of as an infinite-dimensional analogue of minimizing a function over a subset of Euclidean space. This course will present the core techniques of calculus of variations, which are applicable in a wide variety of contexts. We will put particular emphasis on the relationship between variational problems and differential equations, which is useful in both directions: sometimes solving a differential equation helps find the critical points of a functional, and sometimes minimizing a functional helps to solve a differential equation. We will see many examples motivated by geometry and physics, and see how these diverse problems fit into a unified framework.

DYNAMICS: Howard Masur, Kathryn Lindsey, and Aaron Brown

Weeks 1–3:

Title: An introduction to the field of dynamical systems

Abstract: Dynamical systems is a major subject in mathematics that arises in many different contexts. In this minicourse we will give a introduction to the subject focusing on a few basic examples coming from geometry and number theory. We will use these examples to also introduce some important concepts in the subject.

ALGEBRA AND NUMBER THEORY: Frank Calegari, Keerthi Madapusi

Weeks one and two (at least)

Frank Calegari the first week, Keerthi Madapusi the second.

First week: Counting Prime Numbers: We study the problem of how to count prime numbers in various ways, discussing both the prime number theorem and various open problems. We use techniques from both number theory and analysis.

Second week: Lagrange's four squares theorem asserts that every positive integer is the sum of the squares of four integers. It was probably known to Diophantine, and it admits several interesting proofs. I will focus on one using modular forms.

I hope this will continue weeks 3 and 4, and Brandon Levin may talk week 5.

QUADRATIC FORMS, ETC: Shmuel Weinberger

Week 2

Abstract: Quadratic forms are important in many areas of mathematics. After a quick review of the theory of real (and complex) forms and some examples, we will discuss forms over other fields (such as the p-adics – if you know what those are – and rational functions), and over the integers. We will see connections to topology, difference equations on the circle, and Hilbert's 17th problem as well as bizarre sounding facts, like that in any field in which -1 is a sum of 3 squares, it's actually a sum of 2 squares.

LOGIC: Maryanthe Malliaris
Weeks 3-6: Eight talks about the connection between model theory and algebra.

APPLIED: Aditya Khanna
One or two later talks: Computational modeling and epidemiology.

PERSISTENT HOMOLOGY: Jesse Wolfson
Week 1
Topological Data Analysis and Persistent Homology

Lines are frequently used to model data sets (via linear regression), but data may often exhibit richer and more interesting shapes. In this mini-course, I'll explain the ideas behind persistent homology, a recent approach to efficiently uncovering the shape of data. In the process, we will introduce simplicial complexes and their homology groups, which form basic objects of study for modern algebraic topology. No background is assumed.

The more theoretical algebraic topology talks, described below, will give relevant background, theory, and generalizations.

ALGEBRAIC TOPOLOGY: Peter May, Henry Chan, Zhouli Xu

Weeks 1–8. There is a summer school the sixth week which in principal is a separate and more advanced program. See
http://math.uchicago.edu/ chicagotopology2/
http://math.uchicago.edu/ chicagotopology2/program.pdf

We aim to develop enough preliminaries in the first five weeks of the REU that those most interested in the material and willing to work hard can take profit from the summer school. We will try hard to pace the talks slowly enough that those with main interests in other areas can learn from them and maybe even enjoy them. If participants speak up and ask enough questions to slow me down, we may all have fun. I will mentor a few participants who have a special interest in and some prior knowledge of algebraic topology and who plan to write their papers in that subject (interpreted broadly to include category theory and homological algebra). Anyone interested should send me an email. Henry Chan and probably Zhouli Xu will give some of the talks and will arrange problem sessions if asked.