FIFTEEN YEARS OF THE REU AND DRP AT THE UNIVERSITY OF CHICAGO

J. PETER MAY

Contents

Mathematics has become one of the most popular majors at the University of Chicago, now accounting for 10% of all undergraduates. This article describes how that came about, with focus on relevant new programs. A remarkable atmosphere of genuine excitement about mathematics has developed in step with the expansion of these programs. Perhaps only some of the details can be emulated elsewhere, but the crucial idea of collaborating with participants in developing attractive programs can be emulated anywhere.

1. Background and statistics

This article can be viewed as a report on an NSF sponsored experiment.[1] The University of Chicago had two consecutive VIGRE grants, on which I was PI, starting in June of 2000 and continuing until September of 2011. Those grants

[1] It is also an update of a similar article from eight years ago [2].

funded the first twelve years of our REU and several smaller new programs. With considerable internal University of Chicago support, all of the programs initiated with the help of those grants have continued operation the past three years. With the help of a new RTG grant, all will continue at least through 2019. All except the REU appear to be sustainable indefinitely, without external support.

With a focus on undergraduates, we will try to answer what is perhaps the most important question first. What clear, definite, *quantifiable* changes can reasonably be ascribed to the new NSF sponsored programs at Chicago? Only then will we describe what those programs are, how they operate, and how they might be emulated elsewhere. For the last, we will focus on a relatively inexpensive program, the Directed Reading Program (DRP), that can have significant impact.

1.1. **Statistics.** Chicago has a small liberal arts college, with around 5,000 students, embedded in a large research university. Undergraduates have over 100 majors to choose from. The immediate statistical question asks what number and percentage of those students major in mathematics. Here is the answer over time, starting in 2001. That is the last year before the new programs began to have significant impact, and it is also the first year for which we have collected full statistics.[2]

Number and percentage of mathematics majors among graduates, 2001–2014:

	2001	2002	2003	2004	2005	2006	2007
Total	988	947	1,000	1,013	1,072	1,094	1,158
Math	49	57	77	75	76	71	90
Percent	5.0	6.0	7.7	7.4	7.1	6.5	7.8

	2008	2009	2010	2011	2012	2013	2014
Total	1,185	1,207	1,209	1,271	1,242	1,304	1,385
Math	77	96	87	105	103	129	137
Percent	6.5	8.0	7.2	8.3	8.3	9.9	9.9

Visibly, there was a 40% increase in the total undergraduate population over these years and a 180% increase in the total number of mathematics majors. The expansion of the college was planned. Despite the expansion, it appears to us that the quality of undergraduate students has improved significantly over this period. Test scores are corroborative.

It is reasonable to ask how mathematics compares in popularity with other subjects. Here are the top five majors in terms of numbers of bachelor degrees at Chicago over the past eight years:

Subject	2007	2008	2009	2010	2011	2012	2013	2014
Economics	234	211	257	260	298	262	270	320
Biological Sciences	138	134	153	128	170	153	163	203
Political Science	109	111	108	137	148	131	155	140
MATHEMATICS	90	77	96	87	105	103	129	137
English	89	77	88	81	80	92	87	88

One might then ask if this trend is sustainable. The spring count of mathematics majors first passed the 200 mark in 2006, when it reached 227, and it only passed

[2]Roughly 5% of Chicago's undergraduates have been mathematics majors since I arrived in 1967, but time constraints have prevented me from trying to obtain precise earlier data.

the 300 mark in 2011, when it reached 315; it already passed the 400 mark in 2014, reaching 411. Thus, rather alarmingly in terms of our capacity to deal with the numbers, we are on track for still further increases in the number and percentage of mathematics majors in the next few years.

Among U.S. schools with the wealth of options for majors available at Chicago, surely none has percentages and numbers like that. One top university about the same size as Chicago has rightly celebrated an increase to a total of around 40 mathematics majors per year. That is real progress, but it is not on the same scale. Online data and data from graduate admissions[3] show that only MIT has a comparable total number of mathematics majors.

One might then ask about the quality of undergraduate math majors and, specifically, about how many are going on to graduate study at top universities. Virtually all of the top U.S. mathematics departments have current graduate students from Chicago, the vast majority of them having participated in the REU or the DRP. Around 80 students graduating in the past six years, 2009-2014, went on to graduate study in mathematics, attending nearly 40 schools. The most frequent choices were Michigan (13), Berkeley (8), Harvard (4), MIT (4), Stanford (4), Texas (4), Maryland (3), UCLA (3), and Yale (3). Two each went to CalTech, Cambridge, Columbia, Princeton, UIC, and UIUC.

Of these 80, 15 were women. The percentage of women is not nearly as high as we would like, but the number is significant. It includes two winners of the Alice T. Schafer Prize (2010 and 2014). Two other winners (2008 and 2013) were or are Chicago mathematics graduate students.

On average, over 10% of Chicago mathematics majors go on to graduate study in mathematics each year, and over 10% go on to graduate study in other sciences. That only counts those we know are going on to graduate study immediately. A retrospective study of REU participants through 2009 showed that, while fewer than 50% stated they were going on to graduate study at the time they graduated, nearly 80% ended up doing so eventually.

1.2. **The undergraduate program.** What changes accounts for these statistics? Let's start with what did *not* change.

Were there substantial changes to the mathematics curriculum? For better or for worse, the answer is no. Under the pressure of undergraduate demand, courses and tracks have been added, but the basic curriculum has not seen any significant changes, certainly nothing that might seriously affect the statistics. It is worth emphasizing that all undergraduate mathematics courses at Chicago are taught rigorously, with proofs. Some changes made elsewhere were not made at Chicago.

Were there substantial changes in the way mathematics is taught? There has been one significant change, the introduction of inquiry based learning, or IBL, classes (the Moore method). These have definitely had an effect. They are primarily freshman honors calculus classes, which do not provide the fast track for the most advanced incoming students, and most of the honors calculus classes are still taught in the traditional lecture format. The IBL classes do not affect enough students to account for the statistics, but they have had a significant synergistic impact. There have been no other significant changes. In particular, again for better or for worse, Chicago has been backward with respect to technological innovation in teaching.

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[3]As cochair of graduate admissions, I have long kept track of the numbers of applicants to Chicago from all other schools.

Have there been changes of requirements that might have raised enrollment in mathematics classes and might have helped attract mathematics majors? Quite the contrary. There have been some changes in requirements, but these changes have lessened rather than increased the requirements in mathematics. Another change that might well have decreased the numbers is the institution of a new Computer Science major in 2006; there is also a Statistics major.

1.3. **New programs and changes in the culture.** We will focus on two programs with direct impact, the DRP (directed reading program) and the REU. Two other seemingly unrelated programs, the VCA (Vigre Course Assistant) for undergraduates working in our teaching program and the warm-up program for incoming graduate students (WOMP), have an indirect synergistic effect.

This article will have a theme. The key to the increased popularity of mathematics at Chicago is something that is not quantifiable, namely the development of an extraordinary rapport among undergraduates, graduate students, and even postdocs over the past 15 years. Before 2000, these groups were socially separate. Now they are not. Social activities bringing undergraduate and graduate students together are sprinkled throughout the year. For example, just last Saturday (October 25, 2014), there was a “‘math study break” for undergraduates organized by graduate students in our AWM chapter. It was advertised with the enticement that “Math grads will be on hand to help you ~~cheat~~ (!) study for the mid-terms”. For another example, there is an annual and nowadays quite elaborate beer skit (make fun of the faculty — you can see some on the web). It is put on in late spring by second year graduate students. Before 2000, undergraduates had nothing to do with it and rarely if ever attended. Now it is a tradition that undergraduates serve a barbecue before the skits and attend it in droves. We have no room big enough to hold the resulting crowd.

Social intermingling of undergraduate and graduate students also takes place in the quarterly evening talks of the DRP and the occasional lunches and dinners with talks that are a regular part of the REU. Such socializing is intertwined with the mentoring of undergraduates by graduate students that is at the heart of the DRP and REU. Neither program could exist without the melding of levels, vertical integration, to use the slogan, that pervades the mathematics program at Chicago. This plays a central role in both the attractiveness of mathematics to Chicago undergraduates and the success of the graduate program.

A related theme is that our programs are bottom-up rather than top-down. They are driven by student demand, and student demand is driven by the cultural change. In particular, faculty do not recruit undergraduates to our programs. Past participants, graduate students, and postdocs do that job. It is all word of mouth.

2. The DRP and VCA programs

2.1. **Background of the DRP; the VCA.** We will start with the DRP because it is inexpensive and implementable anywhere, or at least anywhere that has a sizable pool of graduate students. In fact, there are current DRP programs that are modeled on Chicago's at seven other schools that we know of; there may be more. It is easy to learn about these programs since they all have websites, listed in the Appendix. It is to be emphasized that these programs and their websites are set up and maintained primarily or exclusively by graduate students. To the best of our knowledge, all of these programs were organized primarily by people who had participated in Chicago's program, either as undergraduate mentorees or as graduate student mentors. They are all based on the same model.

Graduate student initiative led to the DRP, which began in 2002. From the beginnning it has been allied with the role of graduate students as stand-alone teachers in the lower level mathematics courses. The VCA began at the same time, also based on graduate student initiative, and the background is relevant. Chicago has an extensive and complicated system of employment for undergraduates as "Readers" (graders for honors calculus and higher level courses), "course assistants" (graders for the mid level calculus sequence) and "tutors" (teaching activities in the lower level calculus sequence).

When our VIGRE program started, our graduate student teachers were concerned that the course assistant job was less attractive than it should be and was not attracting a suitable caliber of grader. Their ideas for remedying that led to the institution of the VCA program. VCAs are not just graders. Rather they hold office hours and run independent problem sessions additional to those of the graduate student teaching the course. This program has led to markedly fewer complaints about graders from both the graduate student teachers and the undergraduates taking their courses. The increased collaboration of graduate students and undergraduates in the teaching of the lower level courses is one of the synergistic changes that has helped cement good relations between graduate students and undergraduates. The VCA position is now a permanent part of the undergraduate instructional program, funded internally, and we shall say little more about it.

2.2. **How the DRP operates.** Undergraduate mathematics majors want to see more mathematics, and our graduate students want to mentor them. The academic year DRP institutionalizes this. In this program, undergraduates meet one-on-one with graduate students to study some topic of mutual interest, agreed on between the student and mentor. The student learns all that he or she can. Participation is voluntary on the part of both undergraduates and graduate students.

Participants are required to meet with their mentor at least once a week during the quarter and to put in at least four hours of work on their topic per week. They are required to give a presentation at the end of the quarter. Projects must be approved by the committee. The program description advertises the following benefits of the program.

> Participating undergraduates will learn to work independently through studying a topic of their choice, well-suited to their interests. They will develop relationships with graduate student mentors and receive a good deal of personal attention focused on their mathematical studies. Finally, they will gain valuable experience in mathematical communication by giving a presentation on their work to an audience of their peers.

The last point refers to the end of quarter presentations. These are given at evening sessions, dinner provided by the department, attended by the undergraduate participants, their mentors, often some of the undergraduates and mentors who will be participating in the following quarter, and at least one faculty member. The level of the presentations has been uneven but generally very high.

The undergraduates who participate obtain no course credit and are not paid, and they do the work on top of their usually heavy course loads. Many of them are also working to help defray college expenses. The fact that more and more are participating, now averaging around twenty per quarter, argues that the advertised benefits are being delivered. The graduate student mentors are also volunteering their time. They receive some payment (currently $350 per quarter, plus up to $100 for books for the participants, with a little more for the organizing committee). Most other DRPs have a smaller number of participants and lower costs.

We emphasize that the choice of topic is arrived at by the participants themselves, mentee and mentor working together. The program is advertised by fliers around the department. Beyond that, recruitment is by word of mouth.

The program is run by a graduate student committee, under loose faculty supervision. Undergraduates apply to and are screened by the committee. Graduate students volunteer for the program, with recruitment of volunteers by the organizing committee. The committee sets up the mentorship pairings, taking into account the expressed interests in subject matter, perceptions of who would fit well with whom, and often input from from those who have taught the undergraduates. The pairings are subject to faculty approval. The committee is self-sustaining, in the sense that its members recruit new members annually to ensure continuity. In fact, the faculty role is little more than attendance at the required student presentations and regular consultation.

3. The summer REU

3.1. **Background and expansion.** This program has expanded steadily. Our VIGRE proposal requested 20 participants, and there were 22 in 2000, its first year of operation. At the time, even 20 participants seemed ambitious, especially since most REUs were (and are) considerably smaller. We did not intend a program as large as ours is today, and we had no precedent to guide us in scaling up. Quite a few relevant changes were suggested by the participants themselves, including some that led to central features of the current program. The program has evolved and is still evolving. The best piece of advice to anyone starting out as an organizer of any such program is to listen to the participants at all levels and learn from them.

Some years the program has had over 100 participants. Constrained by cost, the last few years have had around 80 participants, nearly all University of Chicago undergraduates, and that seems to be a reasonable steady state. It is natural to ask whether the restriction to University of Chicago students is an essential feature of the program. Opening the program to a few select outside participants is probably desirable and will be implemented in the next few years. However, anything like the present scale would be prohibitively expensive and logistically impractible if opened to wide outside participation. For example, University of Chicago students find their own housing with negligible faculty or administrative input. Graduate students and others in the neighborhood seek to sublet apartments in the summer, and local web sites facilitate this. Finding housing for comparable numbers of outside participants would be a nightmare. Other universities seeking to implement such a large scale program would surely face similar constraints.

3.2. **Organization and finances.** Students are free to follow their own interests. They choose courses, the topic of their research, and the exact mix of courses versus research based on their individual backgrounds and interests. They are assigned graduate student mentors, with whom they must meet at very least twice a week throughout the REU. There are many synergies between the regular academic year program and the REU. Some DRP academic year pairings continue into REU pairings, and some REU pairings continue into DRP pairings the following academic year. We will discuss the main aspects of the program, courses and mentored research, in the following sections.

Websites document the activities of the REU through the years. For the years 2002 through 2011, the address is

`http://www.math.uchicago.edu/~may/VIGRE/VIGREREU2011`

with 2011 changed to the year in question. For 2012-2014, the address is

`http://math.uchicago.edu/~may/REU2014`

with 2014 changed to the year in question. The more recent web pages have detailed course descriptions, hour by hour schedules, and information about the organization of the mentorship pairings. Undergraduate papers are posted on the annual websites, starting in 2006. Since 2007, they are separated into Full program and Apprentice program lists.[4]

There are other activities during the REU about which we will say little. There is an open house, mostly run by graduate students, to offer information and answer questions about going on to graduate study. However, there can easily be too much emphasis on future graduate study. To address this problem, the REU features a well-attended open house at which past participants working in non-academic jobs in the Chicago area answer questions and offer perspectives.

Maximum funding for a full eight week participant is $3,000. There is no additional funding for room and board. Apprentice participants receive just $1,000. Some students receive amounts in between, based in large part on their past academic performance and faculty recommendations. This is very far from ideal. It is a serious concern that we have no formal mechanism for determining need. Students are told that finances are not adequate to fully support all who deserve support. Those not needing support are urged to be generous, and some do choose to forgo support every year. Those in need of additional support are urged to tell the organizer, and every effort is made to make sure that few people, if any, do not participate only for financial reasons.

Graduate student mentors are expected to put in serious effort and are paid $2,000 for the full eight weeks, prorated for lesser periods of participation. While some faculty compensation is desirable, especially for postdocs teaching in the program, the twelve faculty participants in 2014 all taught pro bono. They understood how scarce funding is and were happy to volunteer. In past years, nearly all faculty participants received some small token of appreciation.

Even with that excruciatingly barebones level of individual support, the total cost of the program in 2014 is around $120,000. That may seem expensive, but the total cost per student, just $1,500, is surely lower than that of any other REU.

Aside from insufficient funding for the proper support of participants, perhaps the greatest defect of the REU has been that its organization and administration have been the work of a single faculty member. It is not nearly as difficult to set up such a program as one might imagine, but it does entail significant amounts of time. We emphasize that the actual operation of the program during the eight summer weeks is very much a cooperative effort and requires hardly any organizational or administrative work.

[4]The participant lists are so separated starting in 2004.

3.3. **The 2014 REU.** The program in 2014 had 81 participants, mentored by 27 graduate students (including three seniors going on to graduate study, who helped mentor in the apprentice program). Twelve faculty members taught in the program, five of them senior faculty, two assistant professors, and five Dickson Instructors. The high level of faculty participation is crucial. Participation is of course voluntary, and no great persuasion is needed. Here again word of mouth is critical. Postdocs often volunteer even before being asked to participate, having heard about the program from past participants. Senior faculty are often repeat participants. It is a lot of fun teaching students as committed as those participaing in the REU!

There were over 130 applicants from the University of Chicago in 2014. It is hard to say no to an eager student at the home institution of a program like this, and some students have been allowed to sit in without funding. It is by now accepted that even permission to sit in is a privilege and that we just have to say no to many applicants. There were twelve sit-ins in 2014; they have exactly the same activities as the funded participants, including the requirement to write a paper. We have recruited a few African American and Hispanic students not from the University of Chicago, and that aspect of the program will undergo some future expansion. Recruitment so far has been ad hoc. The African Americans were recruited through the Mellon Mays and Leadership Alliance Programs; I recruited the Hispanic, who contacted me on his own initiative. We intend to learn how to recruit such people more systematically. Students recruited from outside are treated in exactly the same fashion as University of Chicago participants.

The program is only open to students who have just completed their first, second, or third years as undergraduates and who have taken at least the first year honors calculus sequence. (There are two tracks of calculus below that level and several more advanced tracks). The program is advertised in all mathematics classes at the level of honors calculus or above. More precisely, the teachers in those classes, most of whom are postdocs, are told about the program and are asked to actively encourage their stronger students to apply. They also announce to their classes when applications become available. The undergraduate mathematics club also announces that. Word of mouth does the rest.

3.4. **The apprentice program.** One major innovation, forced by demand and the wish to welcome first year students, is a five week "apprentice program", intended primarily for first year students. It has been in operation since 2004, with just a few participants in 2004 and 2005. Formal implementation, with the apprentice program explicitly listed as an option on the application forms, began in 2006. This program has steadily increased in size ever since. There were forty apprentice participants in 2014.

A few of the more advanced first year students, five in 2014, participate from the outset in the full eight week program. All apprentice participants are welcome to stay the full eight weeks, and in 2014 six did so. In recent years quite a few second year students, seven in 2014, have actually preferred to participate in the apprentice program. It serves as an entryway into mathematics for students with newly found interest in the subject.

There is a five week apprentice course, which is attended by all apprentice participants. It is nearly always taught by László Babai. It ranges over many topics, mostly in discrete mathematics of one sort or another. Since this course is so central to the apprentice program, here is the full 2014 abstract:

> The course will develop the usual topics of linear algebra and illustrate them on (often striking) applications to discrete structures. Emphasis will be on creative problem solving and discovery. The basic topics include determinants, linear transformations, the characteristic polynomial, Euclidean spaces, orthogonalization, the Spectral Theorem, Singular Value Decomposition. Application areas to be highlighted include spectral graph theory (expansion, quasirandom graphs, Shannon capacity), random walks, clustering high-dimensional data, extremal set theory, and more.

3.5. **Courses in the 2014 REU.** In addition to the apprentice course, more advanced courses are offered in a variety of areas. They range in duration from one to eight weeks and are taught by senior faculty and postdocs. They are frontloaded in the schedule. Although a few courses continue through the full eight weeks, most conclude earlier. Participants focus on individual work on their papers towards the end of the program; those papers are due by the end of August.

The content of the courses has varied widely from year to year. The more advanced subjects most regularly taught are topics in probability, geometry, topology, number theory, logic, and analysis, all of which had at least short courses in 2014. Abstracts of all courses are on the webpages, but we will give some idea of their content both to give an idea of the kind of mathematics the participants see and to indicate the breadth of faculty participation in the program.

In 2014, Gregory Lawler talked for the first two weeks on two related topics, "Random walk and the heat equation" and "Some challenging models in random walk". Lawler has written a book [1], aimed at undergraduates, that is based on his talks in the REU. Probability continued in weeks three through eight, taught by Antonio Auffinger, who is transitioning from an assistant professorship at Chicago to a tenure track position at Northwestern. His course focused on "Subadditive processes and an introduction to ergodic theory".

There were two consecutive and related three week geometry courses, taught by two Dickson Instructors, Dominic Dotterer and Sebastian Hensel. Two quotations from the abstracts give an idea. The first reads "What can listening to the harmonics of a graph tell us? We will explore ways in which the energy of the overtones quantify the geometric and topological complexity of the graph." The second reads "If a group acts nicely on a space we understand, then we can learn something about the algebra of the group."

Weeks one through four saw a course in number theory focusing on primes of the form $x^2 + ny^2$. It was taught by Matthew Emerton and two Dickson Instructors, Paul Herman and Davide Reduzzi. Like two other recent Chicago instructors, Herman was himself an undergraduate participant in the REU.

There were two one week courses in logic, one taught by Dennis Hirschfeldt on computability and definability and the other by Maryanthe Malliaris, an assistant professor, who managed to tell undergraduates about her celebrated recent solution, with Shelah, of the $p = t$ problem on cardinal invariants of the continuum.

There was a three week analysis course on "Equilibria in nonlinear systems" taught by another Dickson Instructor, Baoping Liu. It was coordinated with a new RTG summer program in analysis, described here:

`http://math.uchicago.edu/chicagoanalysis`.

Some advanced REU participants interested in analysis participated in both the REU and the analysis program. Its two week undergraduate component started a week before the REU, thus minimizing schedule conflicts.

Finally, there was an eight week course on "Finite spaces and larger contexts", taught by May, that being the title of a book in preparation based on his presentations in the REU. Several research contributions by undergraduates have been based on topics in this area, which has the complementary virtues of being easily accessible, seriously interesting, and almost unknown even to experts in algebraic topology. It is a bridge between algebraic topology and combinatorics since finite T_0-spaces are the same thing as finite posets.

The idea behind this wealth of offerings is that students get some glimpse of what mathematics is really about, the great wealth of different directions it has to offer, and, perhaps above all, its interconnectivity. Especially in the first few weeks of the REU, students are encouraged to sample courses. Later in the program, they tend to attend only the courses of most interest to them, and especially courses related to their papers.

3.6. **The required participant paper.** A very major change in the REU was the institution of a required paper, beginning in 2006. This was a suggestion originally made by some of the undergraduate participants. It is now the heart of the program. Papers vary widely in scope, level, and purpose. Some contain original research but most, especially the apprentice papers, are primarily expository. They are written at the levels of the individual participants and vary widely in mathematical sophistication. Students know that their papers will be read and commented on by a senior faculty member, and they know that they will be posted online unless they request otherwise. They take the papers very seriously.

While students are encouraged to follow up questions and topics given during the courses and many of the papers are tied to one or another of the course offerings, others are not. The topics of the more advanced students often run far afield. Students are free to explore any direction, on their own and with mentors. We emphasize that students are urged to work at their own level of comfort.

The role of mentors is crucial. A few of the mentors may be new graduates on their way to graduate school (for the apprentices), a few may be postdocs and other faculty, but most are graduate students. The topics are chosen in consultation with the mentors, and the mentors are there to discuss all stages of the research and writing process. Papers must be submitted to the mentors and must be commented upon by them and revised accordingly before they are submitted to faculty. A senior faculty member reads and comments on all of the papers, and students are expected to revise them before they are posted on the web.

Just as with the DRP, the mentorship pairings are organized by a committee of advanced graduate students. However, in view of the larger scale and scope, these graduate students are recruited from year to year by the organizer of the REU, with whom they work in tandem. Both mentors and mentorees report any problems to the committee (for example, lack of work by a participant or, more frequently, lack of expertise on the part of the mentor in the area of most interest to the mentoree). Mentorship pairings are sometimes changed during the course of the summer as participant interests evolve. Mentors report at least weekly to the committee so that progress can be monitored and problems nipped in the bud, and the committee is in constant contact with faculty.

With 80 participants, individual faculty attention varies, but many of the more advanced participants are directly involved with faculty research. For example, one 2014 paper with four participant authors mentored by Babai finds "The smallest sensitivity yet achieved for a k-uniform hypergraph property", among other new results. To give an idea of the range of topics, we list the 2014 papers at the end. They run the gamut, including serious algebra, geometry, topology, analysis, probability theory, and logic. Some are sophisticated and give useful overviews and perspectives on particular fields. Others are playful. Some are elementary learning experiences. Some contain original research. We reiterate that students are urged to work at their own level of comfort, always mentored, but free to explore any direction. The amount of mathematics the students learn is prodigious. Most important, the students learn to love mathematics. They come to aspire to do mathematics. Many of the students spend two or three summers in the REU. Reading their papers from year-to-year and seeing their growth is eye-opening. Many current graduate students all over the country started out this way.

4. Graduate student participation; the warm-up program

It will by now be obvious that the DRP and REU could not possibly function without dedicated graduate student cooperation. The VCA, DRP, and REU, described above as undergraduate programs are also graduate programs. The supervisors of the VCAs are the graduate students teaching the courses they assist in. The DRP is entirely organized and run by graduate students. The mentoring of undergraduates by graduate students is a central feature of the REU, and that is also organized and run by graduate students.

Graduate student participants in the REU are recruited by email requests for volunteers. This can be delicate. Usually the strongest of the graduate students are the ones who volunteer. However, some graduate students who would like to participate but are finding graduate study more difficult are better off focusing solely on their own research and are advised to do so. It is essential that graduate students genuinely volunteer, with no pressure on them to do so. Finding the right balance in the best interests of both the graduate students and the undergraduates they mentor has been fine tuned over the years. A ratio of three undergraduates per mentor, primarily grouped as six undergraduates under the mentorship of two graduate students, seems to work especially well. Other programs pay their graduate students more and work them harder.

Our programs could not function without an atmosphere of camaraderie and rapport among the graduate students themselves. That is developed from the beginning of their graduate student experience. In fact, it begins even before graduate students arrive. A graduate committee plays a huge role in the recruitment of graduate students, and virtually all graduate students participate in the entertainment of prospective students. In 2000, on their own initiative, graduate students instituted a warm-up program (WOMP) for the incoming class. It is now a permanent part of our program, funded by the Department of Mathematics. Graduate students give summary treatments of material that incoming students will be assumed to know during the first year program. Incoming students get acclimatized before classes begin. They get to know each other and they get to know the role graduate students play in undergraduate education.

The graduate student organizers of WOMP are recruited by faculty and past organizers working together. Essentially as part of their support package, entering students are offered $700 towards support during the two weeks of the warm-up program, and a total of $2500 is allotted to the organizers and the roughly twenty graduate students participating as hosts and speakers in the program.

A crucial feature of all of the programs we have mentioned is the special role that women graduate students play. A quarter of our graduate students are women. That is perhaps not a percentage to be proud of, but it is double the percentage of our peer departments. Virtually all of the graduate committees mentioned have women participants. They make an inestimable contribution to all aspects of the mathematics program.

5. The balance of learning and research

The REU and the DRP both depend on close mentoring, and often joint learning, between undergraduates and graduate students. Students focus on learning and appreciating mathematics, often well beyond what is usually considered the undergraduate level. The students may or may not do research, but they get a taste of it, and they get a serious feeling for the world of graduate mathematics and beyond, serious enough to lead many of them aspire to that as the way to spend their lives.

In both the REU and DRP, rigorous mathematics and a sense of community are combined. In both, the presentations are coupled with dinners and are social occasions. Students often bring their friends. Mathematics is many Chicago students' idea of fun, and it is amazing to see just how contagious a love of mathematics can be. One reason that it is so easy to recruit faculty and graduate students to participate in these programs is that they love the opportunity to teach serious mathematics, rigorously, to younger people who are genuinely interested in learning. The participants appreciate that they are seeing the real thing, mathematics that top flight research mathematicians consider to be important. The combination of a very high level of mathematics and respect for work at any level help to create the congenial atmosphere that make these programs so popular.

What undergraduate research has been undertaken in these programs? There has been some, and some of it has been seriously interesting, but in truth there has not been a great deal in comparison with the number of participants. There is a statement from a 1961 conference on undergraduate research at Carleton College to the effect that "The aims of undergraduate research are the training and stimulation of the student, not the attainment of new results, though such bonuses will come occasionally." I must admit to being a retrograde iconoclast: that does accurately portray the attitude at Chicago. We are very happy to give some of our undergraduates research topics. We are delighted when they prove something new and interesting. We do rope them in by having them work on things we would like to better understand ourselves.

But Chicago is a premier research department. To achieve the level of postdoc and faculty commitment that we have from people at the top of their game, we naturally give complete leeway in what and how they present mathematics to the participants. If they want to focus on undergraduate research, they are free to do so. But they generally don't. To reiterate, ours is not a top down program. We don't tell postdocs, let alone tenured faculty members, how they should run their REU classes. They can and do run them as they please.

It is the most enthusiastic teachers who volunteer, and their energy and enthusiasm are central to the success of the program. We are not focused on undergraduate research per se. We are focused on attracting many of the young people in our care to careers in mathematics and the sciences, and attracting all of them to make mathematics a part of the rest of their lives.

6. Some quotes from 2014 participants

Here are some quotes from 2014 participants. (More quotes may be found in [2].) They are from gratuitous thank yous in emails to which their 2014 papers were attached or from acknowledgments in those papers, and this is just a sampling. I apologize if their inclusion seems immodest. The point is to illustrate the joy people take in the program, which explains why the popularity of the program is contagious. That is the crucial reason the program seems likely to continue.

"Thank you very much for a fantastic Math REU. I gained a new appreciation for high-level mathematics this summer."

"Thank you for running this program; I had a great time, and being completely immersed in math was a fantastic, albeit at times overwhelming, experience."

"Here is my final REU paper. Thank you very much for allowing me to participate and gain meaningful exposure to different disciplines through this program."

"My REU paper is attached. Thank you for a fantastic summer!"

"Thanks so much for all the work you did organizing the program, and for the work you will do reading everyone's papers! I really appreciated the opportunity to learn so much math with so many brilliant people!"

"Thank you for the work you put in to organize the program this year. Over the course of the program, I learned a lot of interesting math and was challenged to think deeply about math and to express my thinking in ways that I have not been before. I feel very fortunate to have been able to participate."

"Thank you so much for running this REU program; I honestly feel like I've learned more math this summer than I ever have, and I would not have wanted to spend my summer doing anything else. It has been a fantastic experience, and I really hope you enjoy my paper!"

"I would also like to thank — for coordinating this wonderful undergraduate research and study opportunity."

"I would like to thank — for making possible the most mathematically productive eight weeks of my life."

"Thanks so much for running the REU this year! It was truly a mathematical blessing."

From an apprentice participant: "Thank you so much for giving me the opportunity to participate in such wonderful program this summer and it would be great if I could return next summer for the full program."

"I also would like to thank — and the University of Chicago Math Department for making this experience possible. It really was wonderful."

"I want to thank — and all of the instructors in the math REU for this wonderful program that helped me to both broaden my knowledge of mathematics and increase my mathematical maturity."

7. Titles of 2014 participant papers

FULL PROGRAM PAPERS

- Calista Bernard. Regularity of solutions to the fractional Laplace equation.
- Joshua Biderman, Kevin Cuddy, Ang Li, and Min Jae Song. On sensitivity of k-uniform hypergraph properties.
- Ben Call. Introduction to Furstenberg's x2x3 conjecture.
- Zefeng Chen. Quasi-preference: choice on partially ordered sets.
- Sean Colin-Ellerin. Distribution theory and applications to PDE.
- Kevin Cuddy. See Joshua Biderman.
- Kyle Gannon. Introduction to the Keisler order.
- Claudio Gonzales. Polynomials in the Dirichlet problem.
- Justin Guo. Analysis of chaotic systems.
- Jackson Hance. Hodge theory and elliptic regularity.
- Jordan Hisel. Addition law on elliptic curves.
- Yifeng Huang. Characteristic classes, Chern classes and applications to intersection theory.
- Sofi Gjing Jovanovska. Beck's theorem characterizing algebras.
- Sameer Kailasa. Topics in geometric group theory.
- Simon Lazarus. Basic algebraic geometry and the 27 lines on a cubic surface.
- Fizay-Noah Lee. Kummer's theory on ideal numbers and Fermat's last theorem.
- Ang Li. See also Joshua Biderman. The Lefschetz fixed point theorem and solutions to polynomials over finite fields.
- Siwei Li. Strategies in the stochastic iterated prisoner's dilemma.
- Jason Liang. Measure-preserving dynamical systems and approximation techniques.
- Lucas Lingle. Intro to class field theory and the Chebotarev theorem.
- Ben Lowe. The local theory of elliptic operators and the Hodge theorem.
- Benjamin McKenna. The type problem: effective resistance and random walks on graphs.
- Redmond McNamara. Introduction to de Rham cohomology.
- Jing Miao. Convergence of Fourier series in L^p space.
- Victor Moros. The zeta function and the Riemann hypothesis.
- Sun Woo Park. An introduction to dynamical billiards.
- Nicholas Rouse. Compact Lie groups.
- Bryan Rust. A theorem of Hopf in homological algebra.
- Maximilian Schindler. Basic Schubert calculus.
- Noah Schoem. Well-foundedness of countable ordinals and the Hydra game.
- Joel Siegel. Expander graphs.
- Maria Smith. Kolmogorov-Barzdin and spacial realizations of expander graphs.
- Min Jae Song. See Joshua Biderman.
- Blaine Talbut. The uncertainty principle in Fourier analysis.
- Victor Zhang. Incompleteness in ZFC.
- Yuzhou Zou. Modes of convergence for Fourier series.

APPRENTICE PROGRAM PAPERS

- Will Adkisson. Geodesics of hyperbolic space.
- Fernando Al Assal. Invitation to Lie algebras and representations.
- John Alhadi. Exploration of various items in linear algebra.
- Kevin Barnum. The axiom of choice and its implications.
- Karen Butt. An introduction to topological entropy.
- Rachel Carandang. Generalization in machine learning: the Vapnik-Chervonenkis inequality.
- David Casey. Galois theory. Plodding, beginner, trying hard, some gaffes.
- Spencer Chan. Compass and straightedge applications of field theory.
- Cindy Chung. An introduction to computability theory.
- Brenden Collins. An introduction to Lie theory through matrix groups.
- Paul Duncan. The Gamma function and the Zeta function.
- Adam Freymiller. Markov chain tree theorem and Wilson's algorithm.
- Michael Hochman. Chutes and ladders.
- Yunpeng Ji. Discussion of the heat equation.
- Ken Jung. Brouwer's fixed point theorem and price equilibrium.
- Josh Kaplan. Binary quadratic forms, genus theory, and primes of the form $p = x^2 + ny^2$.
- James W. Kiselik. Conic and plance curves.
- Daniel Kline. The structure of unit groups.
- Stefan Lance. A survey of primality tests.
- Scarlett Li. Brouwer's fixed point theorem: the Walrasian auctioneer.
- Jack Liang. Rudimentary Galois theory.
- Larsen Linov. An introduction to knot theory and the knot group.
- David Mendelssohn. Operations and methods in fuzzy logic.
- Matthew Morgado. Modular arithmetic.
- Seth Musser. Weakly nonlinear oscillations with analytic forcing.
- Adele Padgett. Fundamental groups: motivation, computation methods, and applications.
- Daniel Parker. Elliptic curves and Lenstra's factorization algorithm.
- Robert Peng. The Hahn-Banach separation theorem and other separation results.
- Peter Robicheaux. Calculation of fundamental groups of spaces.
- Avery Robinson. The Banach-Tarski paradox.
- Zachary Smith. Fixed point methods in nonlinear analysis.
- Aaron Geelon So. Symbolic dynamics.
- Matthew Steed. Proofs of the fundamental theorem of algebra.
- Ridwan Syed. Approximation resistance and linear threshold functions.
- Shaun Tan. Representation theory for finite groups.
- Zachry Wang. Itō calculus and the Black-Scholes option pricing theory
- Nolan Winkler. The discrete log problem and elliptic curve cryptography.
- Eric Yao. Plane conics in algebraic geometry.
- Joo Heon Yoo. The Jordan-Chevalley decomposition.

8. Appendix: web sites of DRP programs

The University of Chicago
`http://math.uchicago.edu/~drp/rails`

The University of California, Berkeley
`http://math.berkeley.edu/wp/drp`

University of Connecticut
`http://www.math.uconn.edu/degree-programs/undergraduate/directed-reading-program/`

The University of Maryland
`http://drp.math.umd.edu/`

MIT
`http://math.mit.edu/research/undergraduate/drp/index.php`

Rutgers University
`http://www.math.rutgers.edu/undergrad/Activities/drp`

The University of Texas
`http://www.ma.utexas.edu/rtgs/geomtop/DRP.html`

Yale University
`http://math.yale.edu/directed-reading-program`

References

[1] Gregory F. Lawler. Random walk and the heat equation. Amer. Math. Soc. 2010.

[2] J.P. May, The University of Chicago's VIGRE REU and DRP. In Proceedings of the conference on promoting undergraduate research in mathematics, pp. 335-340, edited by Joseph A. Gallian. Amer. Math. Soc. 2007.

[3] S. Schuster and K.O. May, Summary and Resolutions, Carleton 1961 Conference. Reprinted in L.J. Senechal, Models for Undergraduate Research in Mathematics, MAA Notes 18, pp 157–159. Math. Asso. of Amer. 1991.