# ABSTRACTS FOR FARRELL'S NAMBOODIRI LECTURES

I. Smooth versus topological rigidity
(Report on joint work with L.E. Jones and W.-C. Hsiang)

Abstract: Mostow proved that n-dimensional closed (compact with empty boundary) Riemannian manifolds $M$ and $N$ with constant $-1$ sectional curvatures and isomorphic fundamental groups are isometric when $n$ is not 2. In the more general setting where the sectional curvatures are negative but not necessarily constant, isometry is clearly too much to expect. But diffeomorphism or at least homeomorphism seemed plausible. We discuss why homeomorphism is true when $n$ is not 4, but diffeomorphism is not in general true.

II. The best of all possible maps
(Report on joint work with L.E. Jones, P. Ontaneda, M.S. Raghunathan and C.S. Aravinda)

Abstract: Eells and Sampson showed that an isomorphism between the fundamental groups of closed negatively curved Riemannian manifolds $M$ and $N$ is induced by a unique (due to Al'ber and Hartman) harmonic map $f$ which is the "best of all possible maps" of the title. In important cases $f$ is a diffeomorphism; e.g. Corlette showed that $f$ is even an isometry (after perhaps scaling) when $M$ is a quaternionic (or Cayley) hyperbolic manifold and $N$ has non-positive curvature operator. It would be wonderful if topological rigidity could be proven this way. Unfortunately the smooth Hauptvermutung result of Siebenmann and Scharlemann intervenes and can be used to produce counterexamples. Also Corlette's result leads to examples of negatively curved manifolds which cannot support a Riemannian metric with non-positive curvature operator.

III. The space of negatively curved metrics
(Report on joint work with P. Ontaneda)

Abstract: Let $R$, $G$, and $T$ denote the spaces of all negatively curved Riemannian metrics, geometries, and marked geometries (respectively) on an $n$-dimensional closed smooth manifold $M$; $G$ and $T$ are quotient spaces of $R$ where isometric and marked isometric metrics (respectively) are identified. We focus attention on the case where $n$ is large instead of the classical setting $n = 2$, and obtain results on the homotopy and homology of $R$, $G$, and $T$; e.g. $R$ has infinitely many components when $n$ is at least 10. And if $M$ supports a real hyperbolic metric (and $n$ is at least 10) then $G$ is also disconnected for sufficiently large covers of $M$. These results motivate the following.

Conjecture. Let $E \to B$ be a smooth bundle whose closed manifold fibers are equipped with negatively curved Riemannian metrics. If $B$ is simply connected, then this bundle is topologically trivial.

We construct many examples which are not smoothly trivial.