Math 176  
Basic Geometry  
Script 1  

January 4, 2010  

1 Euclid: Straightedge and Compass  

Postulates  
Postulates 1. (It is possible) to draw a straight line from any point to any point, and this line is unique.  
Postulates 2. (It is possible) to produce a finite straight line continuously in a straight line.  
Postulates 3. (It is possible) to describe a circle with any center and distance.  
Postulates 4. All right angles are equal.  

Common Notions  
CN 1. Things which are equal to the same thing are also equal to one another.  
CN 2. If equals be added to equals, the wholes are equal.  
CN 3. If equals be subtracted from equals, the remainders are equal.  
CN 4. Things which coincide with one another are equal to one another.  
CN 5. The whole is greater than the part.  

Additional Axioms  
Axiom 1 (SSS). Two triangles are congruent if their corresponding sides all have the same length.  
Axiom 2 (SAS). Two triangles are congruent if two corresponding sides and the angle in between are the same.  
Axiom 3 (ASA). Two triangles are congruent if one corresponding side and the two angles surrounding it are the same.
The Script

Exercise 1.1. Duplicate a line segment.

Exercise 1.2. Given lengths $\alpha$ and $\beta$ with $\alpha > \beta$, construct $\alpha - \beta$.

Exercise 1.3. Construct an equilateral triangle on given side $AB$.

Exercise 1.4. Duplicate an angle.

Exercise 1.5. “Add” two angles.

Exercise 1.6. Bisect angle $POQ$.

Exercise 1.7. Bisect a given line segment $AB$.

Definition 1.8. Two intersecting lines are called perpendicular if the four angles created by their intersection are all congruent. These angles are called right angles.

Exercise 1.9. Construct the perpendicular to a given line $L$ at a point $E$ on the line.

Exercise 1.10. Construct the perpendicular to a line $L$ through a point $E$ not on the line.

Postulates 5 (The Parallel Postulate). Given lines $L, M, N$ and angles $\alpha, \beta, \gamma, \delta$ as shown

Then $\alpha + \beta < \gamma + \delta$ if and only if the lines $L$ and $M$ meet on the right side of the transversal $N$.

Definition 1.11. Two distinct lines are said to be parallel if they never intersect.

Theorem 1.12. Given a line $L$ and a point $P$ not on $L$, there exists a unique line parallel to $L$ through $P$. As part of the proof, give a construction of this line.

Lemma 1.13. Given two parallel lines and any transversal, the alternate interior angles formed by the transversal are congruent.
**Lemma 1.14.** Vertically opposite angles, i.e., angles opposite each other at the intersection of two lines, are equal.

\[
\begin{array}{c}
\alpha \\
\beta \\
\alpha \\
\beta
\end{array}
\]

**Theorem 1.15.** The sum of the angles of any triangle is \( \pi \), the degree of the “angle” at a point between rays that form a straight line.

**Corollary 1.16.** The sum of the angles of any \( n \)-gon is \((n - 2)\pi\).

**Exercise 1.17.** Construct a regular hexagon.

**Exercise 1.18.** Construct a tiling of the plane by equilateral triangles.

**Exercise 1.19.** Construct a tiling of the plane by regular hexagons.

**Exercise 1.20.** Construct a square on a given line segment.

**Exercise 1.21.** Construct a tiling of the plane by squares.

**Theorem 1.22** (Thales). Parallels cut any lines they cross in proportional segments.

**Exercise 1.23.** Divide a line into \( n \) equal parts.

**Exercise 1.24.** Explain why we can’t extend the division of a line into 3 equal parts to the division of an angle into 3 equal parts.

**Exercise 1.25.** Given lengths \( \alpha \) and \( \beta \), construct the length \( \alpha \beta \).

**Exercise 1.26.** Given lengths \( \alpha \) and \( \beta \), construct the length \( \alpha/\beta \).

**Theorem 1.27.** (Converse to Thales) Show that if \( PR \) is not parallel to \( BC \), then \( |AP|/|AB| \neq |AR|/|AC| \).

**Corollary 1.28.** In the figure above, the line \( PR \) is parallel to the line \( BC \) if and only if \( |AP|/|AB| = |AR|/|AC| \).

**Theorem 1.29.** In the figure below suppose \( AB \) is parallel to \( DE \) and \( FE \) is parallel to \( BC \). Then
(i) \(|OA|/|OF| = |OC|/|OD|\) and
(ii) \(AF\) is parallel to \(CD\).

**Theorem 1.30.** Triangles \(ABC\) and \(A'B'C''\) with respective vertices on lines \(L, M,\) and \(N\), as shown in the following diagram, are said to be *in perspective from \(O\).*

If \(AB\) is parallel to \(A'B'\) and \(BC\) is parallel to \(B'C''\), then

\[
\frac{|OA|}{|OC|} = \frac{|OA'|}{|OC''|}
\]

and \(AC\) is parallel to \(A'C''\).

**Definition 1.31.** Triangles \(ABC\) and \(A'B'C'\) are called *similar* if their corresponding angles are equal.

**Theorem 1.32.** Similar triangles have proportional side lengths.

**Lemma 1.33** (Isosceles triangle theorem). If a triangle has two equal sides, then the angles opposite to these sides are also equal.

**Lemma 1.34** (Parallelogram side theorem). Opposite sides of a parallelogram are equal.

**Lemma 1.35.** The diagonals of a parallelogram bisect each other.
Lemma 1.36. The diagonals of a rhombus meet at right angles.

Exercise 1.37. Formulate geometrically and prove the following expression for lengths:

\[(a + b)^2 = a^2 + 2ab + b^2.\]

Theorem 1.38 (Pythagorean Theorem). Given a right triangle with side lengths as shown, then \(a^2 + b^2 = c^2\).