

# Rational billiards, translation surfaces, dynamics

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## 1 Problems

I am going to ask each student to prepare a short lecture on some problem to the class. Here are some possibilities.

1. Show that the Gauss map is ergodic with respect to the Gauss measure.
2. Show that mixing implies ergodicity and the shift map is mixing.
3. Classify all rational billiards that determine a surface of genus 2
4. Show that the regular octagon is in the  $\mathrm{SL}(2, \mathbf{R})$  orbit of an  $L$  shaped table.
5. Suppose that a translation surface  $(X, \omega)$  has a cylinder decomposition where the moduli of the cylinders are rational multiples of each other. Then the  $\mathrm{SL}(2, \mathbf{R})$  orbit of  $(X, \omega)$  contains a product of Dehn twists about the core curves of the cylinders.
6. Show that for any translation surface, for any direction the surface decomposes into flat cylinders and subsurfaces in which each trajectory is dense.
7. Show that for a typical number  $x \in [0, 1)$  the coefficients  $a_i$  of its continued fraction expansion satisfy  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n a_i = \infty$ .
8. Describe a natural  $\mathrm{SL}(2, \mathbf{R})$  invariant measure on each stratum of Abelian differentials and explain why the stratum has finite measure.
9. Describe in detail the Veech group for the regular octagon.