Publication List

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Upper bounds for multiphase composites in any dimension.

In preparation.

We prove a rigorous upper bound for the effective conductivity of a composite made of several isotropic components with given conductivities and volume ratios. The obtained upper bound is in general not achievable and presumably very rough for some values of the parameters. However, it coincides with the Hashin Shtrikman bound when the volume ratio of all phases but any two vanish. It seems to be the first bound for multiphase components with this property in more than two dimensions.

Holder estimates for advection fractional-diffusion equations.

Submitted.

We analyse conditions for an evolution equation with a drift and fractional diffusion to have a Holder continuous solution. The focus of this paper is on the supercritical case, in which we assume that the drift is Holder continuous. The regularity assumptions are the same as in previous works for divergence free drifts, so we obtain the same results without assuming divergence zero.

Holder cont. for int-diff. parabolic eq. with polynomial growth resp. to the gradient.

Discrete and Continuous Dynamical Systems Volume: 28, Number: 3, November 2010. A special issue Dedicated to Louis Nirenberg on the Occasion of his 85th Birthday Part II

We obtain Hölder estimates for an equation of the form

$$u_t + H(x, \nabla u) + (-\Delta)^s u.$$

where $s \in [1/2, 1)$ and H is allowed to have a polynomial growth respect to ∇u of order $2s$. No regularity of H is assumed respect to x.

Eventual regularization of the slightly supercritical fractional Burgers equation.

Joint work with Chi Hin Chan and Magdalena Czubak. Discrete and Continuous Dynamical Systems Volume: 27, Number: 2, June 2010. A special issue Trends and Developments in DE/Dynamics Part I.

We show that even though the weak solutions to the Burgers equation with supercritical fractional diffusion can develop singularities in finite time, the entropy solutions become smooth again after a certain period of time if the exponent of the diffusion is within certain range.

On the differentiability of the sol. to the Hamilton-Jacobi eq. with critical fract. diff.

Advances in Mathematics. To appear.

We study the Hamilto-Jacobi equation with fractional diffusion

$$u_t + H(\nabla u) + (-\Delta)^s u = 0.$$

It was already known that the Caucky problem for this equation has a Lipschitz viscosity solution for any s in the range $s \in (0, 1)$ and a classical solution in the subcritical case $s \in (1/2, 1)$. In the supercritical case $s \in (0, 1/2)$ the solution may become non differentiable regardless of the smoothness of the initial data. In this article we show that the solution is $C^{1,\alpha}$ for the critical case $s = 1/2$. In this case, all terms in the equation are of order one, so no perturbative techniques can be applied.

Smooth approximations to solutions of nonconvex fully nonlinear elliptic equations.

L. Caffarelli and L. Silvestre. American Mathematical Society Translations–Series 2 Advances in the Mathematical Sciences 2010; Volume: 229. Nonlinear Partial Differential Equations and Related Topics: Dedicated to Nina N. Uraltseva.

We show that fully nonlinear elliptic PDEs (which may not have classical solutions) can be approximated with integro-differential equations which have $C^{2,\alpha}$ solutions. This approximation can be used to turn a priori estimates into regularity results. In this paper, we show a $C^{1,\alpha}$ estimate uniform in the approximation. We also study the rate of convergence.

On the Evans-Krylov theorem.

Joint work with Luis Caffarelli. Proceedings of the AMS. 138 (2010), 263-265.

We use the ideas developed for the integro-differential Bellman equation to provide a simplified proof of the Evans-Krylov theorem.

The Evans-Krylov theorem for non local fully non linear equations.

Joint work with Luis Caffarelli. Annals of Mathematics. To appear.

This is the third paper in the series where we develop a regularity theory for fully nonlinear integro-differential equations. We prove that the integro-differential Hamilton-Jacobi-Bellman equation has classical solutions. This is a generalization of the celebrated theorem of Evans and Krylov for convex fully nonlinear elliptic equations. It is interesting to remark that even though there are common ideas, the proof is not an adaptation of the proof in the second order case, but it introduces new ideas even in the local setting.

Regularity results for nonlocal equations by approximation

L. Caffarelli and L. Silvestre. Archive of Rational Mechanics and Analysis. To Appear.

This is the second paper in the series where we develop a regularity theory for fully nonlinear integro-differential equations. We develop the perturbation theory that allows us to obtain regularity results in the variable coefficient case, or in equations that mix either kernels of different orders or drift terms with kernels of order larger than one.

Eventual regularization in the slightly supercritical quasi-geostrophic equation

Annales de l'Institut Henri Poincare (C) Non Linear Analysis 27 (2010), Issue 2, Pages 693-704.

We study weak solutions of the surface quasi-geostrophic equations for supercritical diffusion. We prove that in some range of diffusion exponents the weak solutions become smooth for large time. The proof uses an iterative improvement of oscillation inspired by the work of Caffarelli and Vasseur. In each step of the iteration the obtained improvement of oscillation is used to improve our understanding of the drift term. We have to partially follow the flow of the drift term in order to obtain an extra cancellation that allows us to study a slightly supercritical regime. In that way, more information about the nonlinearity is used than in the work of Caffarelli and Vasseur for the critical case.

The Dirichlet Problem for the Convex Envelope

Joint work with Adam Oberman. Transactions of the AMS. To Appear.

The convex envelope of a given Dirichlet data can be thought of as a degenerate elliptic PDE. We prove some results about this problem. The main result is that if the boundary data is $C^{1,\alpha}$ so is the solution of the equation in the interior, although not necessarily up to the boundary.

Regularity theory for fully nonlinear integro-differential equations.

Joint work with Luis Caffarelli. Communications on Pure and Applied Mathematics. 62 (2009) Issue 5, 597–638.

This is the first of a series of papers where we develop a regularity theory for fully nonlinear integro-differential equations like the ones that arise from stochastic control problems with purely jump Lvy processes. In this paper we obtain an estimate that plays the role of the Alexander-Backelman-Pucci but in the nonlocal setting. Then we develop a Harnack inequality for equations with discontinuous kernels and we use it to obtain $C^{1,\alpha}$ estimates for fully nonlinear equations. It is important that all estimates are independent of the degree of the equations, so the corresponding results for second order elliptic PDE can be recovered as limit cases.

A characterization of optimal two-phase multifunctional composite designs.
Proc. of the Royal Soc. of London A 463, Number 2086 (2007).

We study the problem of optimal design of a two-phase composite material to maximize the sum of thermal and electrical conductivity. We obtain a characterization of an optimal configuration in terms of a free boundary problem. We use our characterization to argue that the optimal interface is not, as has been suggested, a periodic minimal surface. In the process of obtaining this result, we provide a new proof of the relevant case of Bergmans cross property bound. The proof introduces the key idea in the optimal configurations the solutions of the cell problem agree with the directional derivatives of a potential function. This same idea can be used to obtain a very elementary proof of the Hashin-Shtrickman bounds.

Reg. estimatres for the sol. and the free bound. of the obst. prob. for the fract. Lap.
Joint work with Luis Caffarelli and Sandro Salsa. Inventiones Mathematicae. 171, Number 2 (2008).

We use a characterization of the fractional Laplacian as a Dirichlet to Neumann operator to rewrite its obstacle problem as a local free boundary problem. We obtain sharp regularity estimates for this free boundary problem and we also prove the regularity of the free boundary at generic points. The techniques used involve studying the blowup limits using a variation of Almgren's monotonicity formula.

Regularity for the nonlinear Signorini problem.
Joint work with Manolis Milakis. Advances in Mathematics. 217, Issue 3 (2008).

We prove that solutions to the thin obstacle problem with arbitrary convex fully nonlinear equations are $C^{1,\alpha}$ for some small $\alpha > 0$.

An extension problem related to the fractional laplacian.
In collaboration with Luis Caffarelli. Communications in Partial Differential Equations, 32 (2007) 8, 1245.

We characterize the fractional Laplace operator $(-\Delta)^s$ for any power $s \in (0, 1)$ as the Dirichlet to Neumann operator for a degenerate elliptic equation in one more dimension. The elliptic equation can be thought of as a Laplace equation in fractional dimension, and this intuition leads to several useful formulas. The characterization is used to translate nonlocal problems involving fractional diffusion into local problems in one more dimension and therefore it allows us to apply a different type of techniques.

Regularity of the obstacle problem for a fractional power of the laplace operator
Communications on Pure and Applied Mathematics. 60 (2007), no. 1, 67–112.

The solution of the obstacle problem for the operator $(-\triangle)^s$ and an obstacle ϕ is the least function u such that $u \geq \phi$ and $(-\triangle)^s u \geq 0$. In this work it is shown that such solutions are in the space $C^{1,\alpha}$ for every $\alpha < s$. These operators arise in stochastic theory and are related to Levy processes. The obstacle problem is used in financial mathematics to model pricing of american options. When $s = 1/2$, the problem is equivalent to the Signorini problem. It was shown very recently that solutions of the Signorini problem are $C^{1,1/2}$.

Issues in homogenization for problems with nondivergence structure.
In collaboration with Luis A. Caffarelli. Calculus of variations and nonlinear partial differential equations, 43–74, Lecture Notes in Math., 1927, Springer, Berlin, 2008.

These are the Lecture notes of Caffarelli's lectures in the CIME course Calculus of variations and non linear partial differential equations (http://web.math.unifi.it/users/cime/Courses/2005/02.php).

Hölder estimates for solutions of integro differential equations like the fractional laplace
Indiana University Mathematical Journal 55 (2006), 1155-1174.

In this paper there is a purely analytical proof of Hölder continuity for harmonic functions with respect to a class of integro differential equations like the ones associated with purely jump processes. There are some previous proofs but they are probabilistic and their assumptions are not as flexible. Our assumptions include the case of an operator with variable order, without any continuity required for that order.

Regularity for fully nonlinear elliptic equations with Neumann boundary data

In collaboration with E. Milakis. E. Communications in Partial Differential Equations 31 (2006), No. 8

In this paper we study the regularity up to the boundary for viscosity solutions of fully nonlinear equations with Neumann type boundary conditions. Hölder continuity results are obtained for the first and second derivative of the solutions. For the second derivative bound, it is required that the equation is convex or concave.

The two membranes problem

Communications in Partial Differential Equations 30 (2005), no. 1-3, 245–257

This paper concerns the regularity of the free boundary in a problem that models two membranes in contact with each other. Solutions of the problem are obtained as the minimizers of a functional depending on two functions u and v with the constraint that $u \geq v$ and some boundary condition. The functions u and v satisfy a second order partial differential equation in the set where they do not touch. The regularity result for the free boundary is the same as in the usual obstacle problem.

Weak Matrix Majorization

In collaboration with F. D. Martínez Pería and P. Massey. Linear Algebra Appl. 403 (2005), 343–368. joint work with Francisco Martnez Pera and Pedro Massey

This is a paper in linear algebra that compares different ways to extend the notion of majorization to matrices. Under extra conditions it is shown when these notions are equivalent.