

**Math 163, Section 33**  
**Honors Calculus, III (Spring 2008)**  
**Midterm 2**  
(Wednesday May 28).

**Part A: Theory**

PART A1: DEFINITIONS AND THEOREMS

1. State the fundamental theorem of algebra

2. Give the definition of vector subspace of a vector space  $V$  over a field  $F$

3. Give the definition of linear independence and spanning set for a finite set of vectors  $v_1, \dots, v_n$  of a vector space  $V$  over a field  $F$ .

4. Give the definition of  $F$ -linear map between two vector spaces over a field  $F$

PART A2. QUESTIONS: TRUE OR FALSE?

1. Any polynomial of degree  $> 0$  with coefficients in  $\mathbb{R}$  has a real root. (T) (F)
2. If a polynomial  $P(z)$  with coefficients in  $\mathbb{C}$  has a complex root  $w$  (that is,  $P(w) = 0$ ), then also  $\bar{w}$  is a root (that is  $P(\bar{w}) = 0$ ). (T) (F)
3. The field of complex numbers is an ordered field (T) (F)
4.  $\mathbb{Z}$  is a field (T) (F)
5. If  $S = \{u_1, \dots, u_n\}$  is a spanning set for the vector space  $V$  over  $\mathbb{R}$  there exists a subset of  $S$  which is a basis. (T) (F)
6. Consider the vector space  $\mathbb{R}^2$ . Consider the vector subspace  $W = \{(x, y) \in \mathbb{R}^2 \mid a_1x + b_1y = 0, a_2x + b_2y = 0\}$  where  $a_1^2 + b_1^2 \neq 0$ ;  $a_2^2 + b_2^2 \neq 0$ . Then  $W$  is of dimension 0, that is,  $W = \{0\}$ . (T) (F)
7. If  $f : V \longrightarrow W$  is a linear map between two finite dimensional vector spaces then  $\dim \ker f = \dim f(V) - \dim V$ . (T) (F)
8. Any non empty subset of a linearly independent set of vectors in a vector space is linearly independent (T) (F)
9. If  $V$  is a vector space over  $\mathbb{R}$  then  $V$  is a vector space over  $\mathbb{C}$  (T) (F)
10. If  $f : \mathbb{C} \longrightarrow \mathbb{C}$  is a complex function, then  $f$  is continuous if and only if  $f + \bar{f}$  and  $f - \bar{f}$  are continuous. (T) (F)
11. The complex conjugate :  $z \longmapsto \bar{z}$  is a  $\mathbb{C}$ -linear map from  $\mathbb{C}$  to  $\mathbb{C}$ , where we see  $\mathbb{C}$  as a vector space over itself of dimension 1. (T) (F)
12.  $\mathbb{R}$  is a vector space over  $\mathbb{Q}$ , of finite dimension. (T) (F)
13.  $\mathbb{C}^2$  is a real vector space of dimension 2 over  $\mathbb{R}$ . (T) (F)

### Part B: Exercises

**Exercise 1.** Consider the map between the vector spaces:  $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ , defined by:

$$f(x, y, z) = (2x + y + 3z, -x + 3y - 5z, x + 2y) .$$

- Show that it is  $\mathbb{R}$ -linear.
- Find a basis for  $\ker f$  and for  $f(V)$ .
- Find  $\dim \ker f$ ;  $\dim f(V)$ .



**Exercise 2.** Let  $V$  a vector space over  $\mathbb{C}$  of dimension  $n$ . Then  $V$  is a vector space over  $\mathbb{R}$ . Compute  $\dim_{\mathbb{R}} V$  (the dimension of  $V$  as a real vector space). (Hint: take a basis  $v_1, \dots, v_n$  of  $V$  over  $\mathbb{C}$ . Show that  $v_1, iv_1, \dots, v_n, iv_n$  is a basis for  $V$  over  $\mathbb{R}$ .)



**Exercise 3.** Find all the solutions of the equation  $z^4 = 2 + i$

