Math 163, Section 51
Honors Calculus, 3 (Spring 2008)
Homework 7
(due Friday, May 25).

Exercise 1. Say if the following subsets $W$ of the respective vector spaces $V$ are vector subspaces or not (and motivate why). If yes, say if they are finite dimensional. If yes, compute the dimension.

- $V = (\mathbb{R}^3, +, (0,0,0))$ (over $\mathbb{R}$); $W = \{(x, y, z) \mid 3x + 2y + z = 0\}$.
- $V = (\mathbb{R}^3, +, (0,0,0))$ (over $\mathbb{R}$); $W = \{(x, y, z) \mid y + 4z = 0; y - 2x - z = 0; y - x + 3z = 0\}$
- $V = (\mathbb{R}^3, +, (0,0,0))$ (over $\mathbb{Q}$); $W = \{(x, y, z) \mid x + 2y = 0; 3x - z = 0\}$
- $V = \mathbb{R}^R$ (over $\mathbb{R}$); $W = \{f \in C^1(\mathbb{R}, \mathbb{R}) \mid 2f + f' = 0\}$
- $V = \mathbb{Q}[x]$ over $\mathbb{Q}$; $W = \{f \in \mathbb{Q}[x] \mid \deg f \leq 3, f(0) = 1 = f(1)\}$

Exercise 2. Prove that the following maps $f : V \rightarrow W$ between the following vector spaces are linear maps over the field $F$. In the case $V$ and $W$ are finite dimensional, find $\ker f$ and $f(V)$ and compute their dimension.

- $V = W = \mathbb{R}^3$, $F = \mathbb{R}$; $f(x, y, z) = (z, 2x + y, 3x - 2z)$
- $V = W = \mathbb{R}^3$, $F = \mathbb{R}$; $f(x, y, z) = (2x + y + z, x + y + 2z, x + 2y + 5z)$
- $V = \mathbb{R}^3$, $W = \mathbb{R}^2$; $F = \mathbb{R}$; $f(x, y, z) = (x + z, y - x)$
- $V = \mathbb{R}^3$, $W = \mathbb{R}^2$; $F = \mathbb{R}$; $f(x, y, z) = (x + z, -x - z)$
- $V = C^0([a, b], \mathbb{R})$, $W = \mathbb{R}$; $F = \mathbb{R}$; $f(g) = \int_a^b g$
- $V = C^1([a, b], \mathbb{R})$, $W = C^0([a, b])$; $F = \mathbb{R}$; $f(g) = g'$
- $V := \{f \in \mathbb{Q}[x] \mid \deg f \leq 3\}$, $W = \mathbb{F}$, $F = \mathbb{Q}$; $f(p) = p'$

Exercise 3. Let $V$, $W$ vector fields over a field $F$. let $f : V \rightarrow W$ a $F$-linear map between them. Show that if $E$ is a subfield of $F$, then $f$ is $E$-linear.

Prove that the converse is not true, exhibiting a field $F$ and a subfield $E$ and a $E$-linear map $f : V \rightarrow W$ bewteen two vector spaces $V$ and $W$ such that $f$ is not $F$-linear.

Exercises 1, 2(i)-(iv), 3, 4, 5, 6 from Spivak, chapter 25