Exercise 1. Let \( f : \mathbb{R}^3 \to \mathbb{R} \) a \( C^1 \) function, and consider the differential form

\[
\omega = df = \frac{\partial f}{\partial x_1} \, dx_1 + \frac{\partial f}{\partial x_2} \, dx_2 + \frac{\partial f}{\partial x_3} \, dx_3 .
\]

Show that for any \( C^1 \) path \( \gamma : [a, b] \to \mathbb{R}^3 \) such that \( \gamma(a) = p, \, \gamma(b) = q \), we have:

\[
\int_{\gamma} \omega = f(q) - f(p) .
\]

Consider the differential form: \( \omega = 2xy \, dx + x^2 \, dy + dz \). Prove that for any \( C^1 \) curve \( \gamma : [a, b] \to \mathbb{R}^3 \), \( \gamma(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t)) \), we have:

\[
\int_{\gamma} \omega = \gamma_1(b)^2 \gamma_2(b) - \gamma_1(a)^2 \gamma_2(a) + \gamma_3(b) - \gamma_3(a) .
\]
Exercise 2. Consider the parameterized curve:

\[ \gamma : [-1, 1] \rightarrow \mathbb{R}^2 \]
\[ t \mapsto (1 - t^2, t + t^2) \]

- Show that \( \gamma \) is a smooth curve and write the tangent line in any point.
- Consider the vector field \( F := ye_1 + e_2 \) on \( \mathbb{R}^2 \). Compute the integral
  \[ \int_{\gamma} F \cdot T. \]
- Can you find a potential for \( F \), that is, a function \( f \) such that \( \nabla f = F \)?
Exercise 3. Consider the parametrized surface:

\[ \varphi : B(0, 1) \longrightarrow \mathbb{R}^3 \]

\[ (u, v) \longmapsto (u, 1 - u^2 - v^2, v) \]

where \( B(0, 1) \) is the ball in \( \mathbb{R}^2 \), centered at the origin and of radius 1.

- Draw the surface.
- Prove that \( \varphi \) is a smooth surface.
- Compute the surface area \( \sigma(\varphi) \).
- Find an equation for the tangent plane in any point.
- What is the boundary of this surface?