Exercise 1. Take $E$ a Jordan region in $\mathbb{R}^n$. Consider the changement of coordinates:

$$\varphi(x_1, \ldots, x_n) = (\lambda_1 x_1, \ldots, \lambda_n x_n),$$

for all $i = 1, \ldots, n$, with $\lambda_i \in \mathbb{R}$, $\lambda_i \neq 0$. Prove that $\varphi$ is a diffeomorphism of class $C^\infty$ on all $\mathbb{R}^n$ and that $\text{Vol}(\varphi(E)) = \text{Vol}(E)|\lambda_1| \cdots |\lambda_n|$.

Exercise 2. Find a formula for the volume of the $n$-dimensional ball $B_n(r) \subset \mathbb{R}^n$ ($n$-dimensional ball of radius $r$, centered in $0$), in the following steps:

- Reduce the computation of $\text{Vol}(B_n(r))$ to the computation of $\text{Vol}(B_n(1))$, thanks to the previous exercise.
- Use Fubini theorem to prove the following formula:
  $$\text{Vol}(B_n(1)) = \int_0^\pi \text{Vol}(B_{n-1}(\sin \theta)) \sin \theta \, d\theta.$$  
  (Hint: integrate by big slices...)
- Deduce a recursive formula for $B_n(1)$ in terms of $B_{n-1}(1)$ and a computable coefficient $C_n$, given in term of an integral of some function $f_n$ and depending only on $n$.
- Deduce from the previous point a formula for $B_n(r)$ in terms of the coefficients $C_k$, $0 \leq k \leq n$.
- By integration by parts find a recursive relation of $C_n$ in terms of $C_{n-2}$; compute $C_0$ and $C_1$, then compute all the terms $C_n$.
- Deduce a general explicit formula for $\text{Vol}(B_n(r))$.

Exercise 3. Combining exercise 1 and 2, find immediately the volume of the ellipsoid

$$\{(x_1, \ldots, x_n) \in \mathbb{R}^n \mid \sum_{i=1}^n \frac{x_i^2}{a_i^2} \leq r^2\}$$

Exercise 2, 3, 4 in Wade, chapter 12.4, proving first of all that the regions of integrations are Jordan regions, than that the function is integrable, then motivating in detail a changement of variables (if necessary).

Exercise 6, 9, 10 Wade chapter 12.4.

\[\text{but don’t compute it...}\]