Mathematics 16100
Problem Set #6
Due November 9, 2006

Throughout this problem set, $(C, <)$ satisfies Axioms 1 and 2.

1. Suppose $A, B \subset C$. Prove that
$$\overline{A \cap B} \subset \overline{A} \cap \overline{B}.$$
Can one expect equality in general? Why or why not?

2. If $A \subset C$, we define the interior $int(A)$ to be the largest open set contained in A, i.e. $int(A) \subset A$ is open (as a subset of C), and any other open subset $B \subset A$ is contained in $int(A)$. Show that any set $A \subset C$ has an interior.

3. For any subset $A \subset C$, we define the boundary $bd(A)$ to be the set
$$bd(A) = \overline{A} \cap \overline{C \backslash A}.$$
   (a) Show that $bd(bd(A)) \subset bd(A)$.
   (b) If A is closed, show that $bd(bd(A))) = bd(A)$. Is this still true if we don't assume that A is closed? Why or why not?

4. Show that the set $A = \{x \in \mathbb{Q} | x^2 < 2\}$ is both closed and open as a subset of $\mathbb{Q}$.