Mathematics 16100
Problem Set #4
Due October 26, 2006

1. Prove that any nonempty subset of $\mathbb{N}$ has a smallest element with respect to the usual ordering. (This property is often expressed by saying that $\mathbb{N}$ is well-ordered.)

2. A set is said to be countable if it is either finite or countably infinite. Show that any subset of a countable set is again countable.

3. (a) If X is a set, let $P(X)$ denote the set of all subsets of X. Suppose that there were a one-to-one correspondence Y between X and $P(X)$. Define a set B by

$$B = \{a \in X | (a, A) \in Y, \, a \notin A\}$$

Show that $B \subset X$ but that there is no $b \in X$ such that $(b, B) \in Y$.

(b) Prove that there does not exist a one-to-one correspondence between X and $P(X)$ for any set X.

(c) Prove that $P(\mathbb{N})$ is not countable.