Mathematics 16100
Problem Set #1
Due October 5, 2006

1. Show that
$$1 + 3 + 3^2 + \ldots + 3^{k-1} = \frac{3^k - 1}{2}$$
for every $k \in \mathbb{N}$. Give two proofs of this: one by induction and one not.

2. Show that any natural number $n \geq 22$ can be written as $3k + 11l$ where k and l are both whole numbers.

3. Show that there are no natural numbers m, n satisfying the equation $m^2 - n^2 = 14$.

4. Show that there are infinitely many natural numbers $p \in \mathbb{N}$ such that there are no natural numbers m, n satisfying $m^2 - n^2 = p$.