TEACHING PHILOSOPHY

My teaching philosophy has been shaped by the many ways I interact with students. I have worked alongside them as a volunteer tutor at an elementary school, I have guided them through independent study projects in the University of Chicago's directed reading and REU programs, and I have been their instructor. As a technical assistant in one of Paul Sally's Inquiry-Based-Learning classes[1], I have witnessed students discovering theorems, mistakes in proofs and forging interesting questions. These experiences have taught me to not underestimate the ability of my students to find their way once the path has been laid out in front of them. While I do not believe the Inquiry-Based-Learning approach is appropriate for all students, I do believe that, with an appropriate amount of guidance, students can uncover the inner-workings of mathematics on their own.

Through carefully crafted problem sets, I have my students work through lemmas that I will need in a future lecture or discover some of the easier results that would normally be covered in class. Giving the former type of problems help keep my students engaged when I am lecturing on a difficult or technical topic; and the latter problems can give my students a deeper understanding of the material, while simultaneously giving me additional time to cover other material. Both types of problems help bring my students into the center of the learning process and show them how mathematics works.

For example, when covering the fundamental theorem of calculus, I will give my students a sequence of problems studying the function

$$L(x) = \int_1^x \frac{dt}{t}$$

(most of my students have not seen logarithms at this point and I ask those that have to not identify $L(x)$ with $\ln x$). The sequence of problems that follow allow them to determine all of the properties of the logarithm and illustrate the need for specific theorems and definitions from class; here are some sample exercises:

Demonstrate the following:

(1) $L(1) = 0$ (Definition of the integral)
(2) $L(a \cdot b) = L(a) + L(b)$ (Fundamental theorem of calculus, if two functions have the same derivative they must differ by a constant)
(3) $L(2^n) \geq \frac{n}{2}$ (Lower sums)
(4) $L(x)$ is monotonically increasing (Fundamental theorem of calculus and properties of the derivative)
(5) There exists a unique number $e \in [1,4]$, such that $L(e) = 1$. (Intermediate value theorem and the injectivity of monotonic functions)

For my first year students each statement is broken down into a sequence of exercises requiring at most two steps.

I like this sequence of problems for several reasons: a variety of tools are necessary to solve them, they are reasonably easy, and students have some geometric intuition on what the integral represents to guide them. At the same time, these problems familiarize students with the process of using definitions and theorems while building up their confidence in writing proofs. These problems naturally lead to others such as properties of the exponential function and the irrationality of e. More importantly, students see that the logarithm function is not constructed from thin air with certain prescribed properties, but that it arises naturally and its properties are built up methodically.

The transition from simple calculations to more thoughtful questions is a difficult one. To help students move through it, I feel it is extremely important to be aware of how they are progressing. Through office hours, problem sessions and class interaction I try to identify what my students find confusing and if the pace is appropriate. After the first exam, I meet with each of my students for a few minutes to identify why they missed certain problems. This allows me to accurately assess their abilities, identify if many of them suffer from the same sort of confusion, and remind them of the support I can offer. For example, through these meetings I identified that my treatment of induction had been too informal for some of my students and that, for these students, a more axiomatic approach clarified the process. This new approach saved both my students and myself hours of frustration in and out of the classroom.

To teach this way, I spend a disproportionate amount of time structuring problem sets and, early on, meeting with students. The class moves slowly at first, but once students have acclimated, they quickly learn new material. After five weeks, I witness a remarkable growth in their abilities and their enthusiasm. While they still stumble on the language, they are able to grasp the fundamental ideas and understand what is being done. Many students are able to recognize that the same idea underpins different topics. Some begin to understand that mathematics is not the art of pushing symbols around the page and building abstraction for abstraction's sake, but a science in which experiments can be done with pencil and paper and that abstraction is necessary to describe the results.

[1]This is a Moore-method course. There are no books or lectures, students are given a well-constructed list of definitions and theorems and asked to prove them before the class. The teacher rarely intrudes on the discussion as the students try to find a correct proof.