1. (10 points)

Let

\[
A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.
\]

Define \( L_A : \mathbb{R}^2 \to \mathbb{R}^2 \) as the matrix multiplication by \( A \) on the left

\[
\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}
\]

1) Let \( \vec{e}_1 = (1, 0) \) and \( \vec{e}_2 = (0, 1) \), which we think as column vectors.
Please compute \( L_A(\vec{e}_1) \) and \( L_A(\vec{e}_2) \), and write them as column vectors.

2) Please show that \( L_A \) is linear.

i.e. For \( \vec{x}, \vec{y} \in \mathbb{R}^2 \), and \( \lambda \in \mathbb{R} \), we have \( L_A(\vec{x} + \lambda \vec{y}) = L_A(\vec{x}) + \lambda L_A(\vec{y}) \).
2. (10 points)
Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $F(x_1, x_2) = (x_1x_2, x_1^2 + x_2^2)$.
Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $G(x_1, x_2) = (x_2, 2x_1)$.

1) Please first compute the derivatives $dF_{(1,2)}$ and $dG_{(1,1)}$.
Then compute the product $dF_{(1,2)} \cdot dG_{(1,1)}$.
(Hint: all the answers should be two-by-two real valued matrices.)

2) Please compute the expression for $F \circ G : \mathbb{R}^2 \to \mathbb{R}^2$,
i.e. write down the formula for $F(G(x_1, x_2))$.
Then compute its derivative at $(1, 1)$,
i.e. compute the two-by-two matrix $d(F \circ G)_{(1,1)}$. 