LESSON 2: GRAPHS OF FUNCTIONS

Keywords
graph, dependent variable, independent variable, coordinate plane, abscissa, ordinate, even function, odd function

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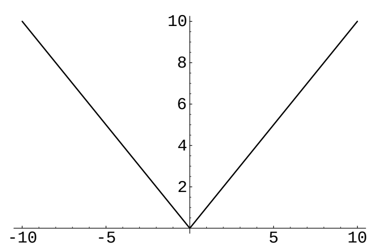
Sometimes we can write a function as $y = f(x)$. Then x is called the *independent variable* and y is called the *dependent variable* (because the value of each y will depend on our choice of the input x). For example, writing $y = x^2 - 2x + 4$ is the same as writing $f(x) = x^2 - 2x + 4$.

When both the domain and range of a function are sets of real numbers, we can picture the function by drawing its graph on a coordinate plane. A coordinate plane is formed by two perpendicular lines (called *axes*), where the horizontal line is called the *abscissa* and is used always for the *input* variable. The vertical axis is used for the *output* or *dependent* variable, and it is called the *ordinate.*

Here is how the graph of the absolute value function looks:



Absolute Value Function

Due to the definition of a function, we *cannot* have two different outputs for a given input. To tell whether a graph is the graph of a function, we draw vertical lines. If a vertical line goes through two points of the graph, it means that, for a fixed input x, the association gives two different outputs y_1 and y_2, and the graph cannot be the graph of a function. Therefore, if no vertical line cuts the graph at more than one point, then the graph is the graph of a function (see Figure 2.1).

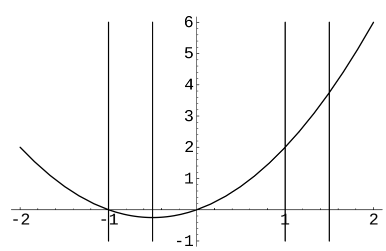


Figure 2.1

How do we decide whether a function is injective just by looking at its graph? We draw horizontal lines. If a horizontal line cuts the graph at more than one point, this means that there are two different values of x that give the same value y, hence the function is *not* injective. If no horizontal line cuts the graph at more than one point, then the corresponding function is injective (see Figure 2.2).

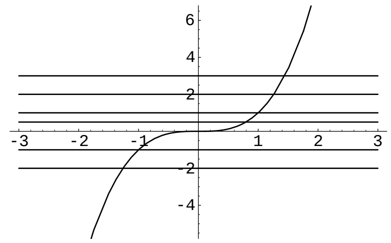
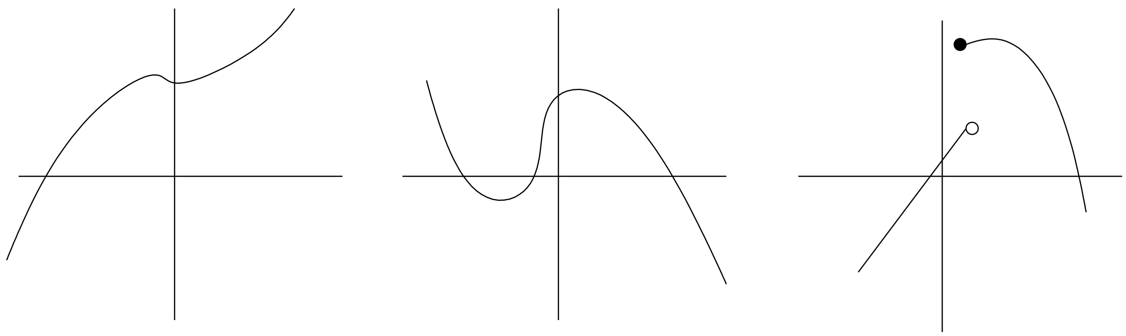
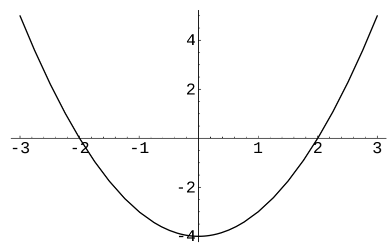


Figure 2.2

Examples of graphs of functions:



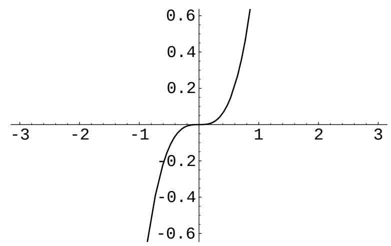
An even function is a function f such that $f(x) = f(-x)$ for all real numbers x. For example, $f(x) = x^2$ is an even function (because $f(-x) = (-x)^2 = x^2 = f(x)$), while $g(y) = y - 2$ is not an even function (this can be easily checked by noting that $g(2) = 0$ and $g(-2) = -4$ and hence $g(2) \neq g(-2)$). The graph of an even function is *symmetric with respect to the y-axis* (or, in other words, the vertical line of the coordinate plane should work as a mirror for this graph). Here is the graph of $f(x) = x^2 - 4$, which is an even function.



$f(x) = x^2 - 4$

Check that the absolute value function is an even function!

An odd function g is a function such that $g(-x) = -g(x)$ for all real numbers x. One simple example of an odd function is the function $h(y) = y$. Notice that $h(-y) = -y = -(y) = -h(y)$. The graph of an odd function should be symmetric with respect to the origin of the coordinate plane (the origin is where the two axes intersect). The graph of $g(x) = x^3$, an odd function, is shown below.

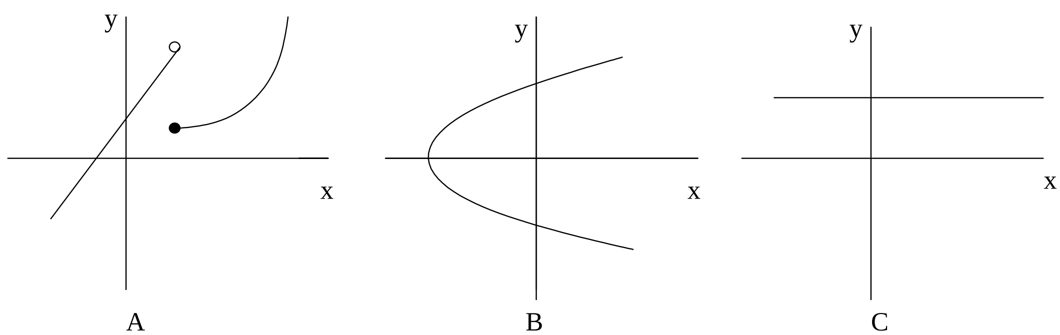


$g(x) = x^3$

Exercises

1. Which of the graphs below are graphs of functions?

   (a) B and C (b) A only (c) A and C (d) all of them



Exercise 1

2. Which of the functions below is an even function?

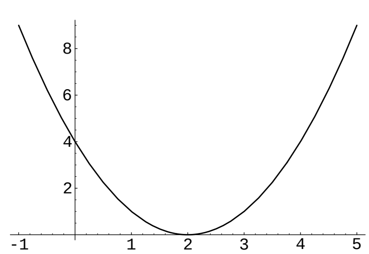
   (a) $f(x) = x^3$ (b) $H(x) = x$ (c) $g(x) = x^4 - 3$ (d) $m(x) = (x - 2)^2$

3. The function $f(x) = -|x|$ is an even function.

   (a) True (b) False

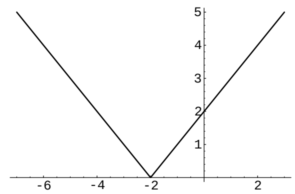
4. The graph of $f(x) = x^2 - 4x + 4$ is as shown below.

   (a) True (b) False



Exercise 4

5. The graph of $f(x) = |x - 2|$ is as shown below.
(a) True (b) False



Exercise 5

Solutions

1(c) (even though the graph "breaks" at (A), if we draw a vertical line going through every point, each line only intercepts the graph at at most one point) 2(c) ($f(x)$ and $H(x)$ are odd functions, and $m(x)$ is neither odd nor even. It is easy to check that $m(x)$ is not even: $m(2) \neq m(-2)$) 3(a) 4(a) 5(b) (if $f(x) = |x - 2|$, then $f(2) = 0$, which is not true for this graph)