Problem 1 Decide for the following subsets of \( \mathbb{R} \) if infimum, supremum, minimum, and maximum exist and determine them where appropriate.

- \( M_1 = \{ x \in \mathbb{R} : -1 \leq x^2 \leq 3 \} \)
- \( M_2 = \{ x \in \mathbb{Q} : x^2 \leq 3 \} \)
- \( M_3 = \{ x \in \mathbb{Z} : x^2 \leq 3 \} \)
- \( M_4 = \{ x \in \mathbb{R} : x^3 < 27 \} \)

Problem 2

(i) Spivak 8.13

(ii) For \( A, B \subset \mathbb{R} \) define \( A \cdot B = \{ ab : a \in A, b \in B \} \) What can you say about \( \text{sup}(A \cdot B) \) if \( A \) and \( B \) are non-empty and bounded from above?

Problem 3 Let \( f : [0, \infty) \to \mathbb{R} \) be a continuous function, such that

\[ \forall \varepsilon > 0 \; \exists C > 0 \; \forall x > C : |f(x)| < \varepsilon. \]

Show that \( f \) attains its maximum or minimum.

Problem 4 Spivak 8-A.1

Problem 5 Spivak 8-A.2

Problem 6 Spivak 9.8

Problem 7

(i) Spivak 9.14

(ii) Spivak 9.15

Problem 8 Spivak 9.19

Problem 9* (extra credit) Prove or give a counterexample to the following statement: Let \( \varepsilon > 0 \) and \( f : \mathbb{R} \to \mathbb{R} \) be uniformly continuous with \( f(0) = 0 \). Then there is \( C > 0 \), such that \( |f(x)| \leq \varepsilon + C|x| \) for any \( x \in \mathbb{R} \).