ÉTALE (φ, Γ)-MODULES AND OVERCONVERGENCE

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Abstract. These are notes for a talk which introduces étale (φ, Γ)-modules and overconvergence. We present the concept and describe some of its applications, with an emphasis on examples.

This is an introductory note concerning étale (φ, Γ)-modules and overconvergence. For simplicity, we will only consider the case where the base field K is a finite unramified extension of $\mathbb{Q}_p$ for some prime p. Topological issues will often be ignored; in particular any representation, action or endomorphism is assumed to be continuous.

1. Étale φ-Modules

Let $\mathbf{E}_K = k((X))$ where k is the residue field of K, and let $G_{\mathbf{E}_K}$ be its absolute Galois group. Consider the ring

$$\mathbf{A}_K = \left\{ \sum_{n=-\infty}^{\infty} a_n X^n,\ a_n \in \mathcal{O}_K,\ a_{-n} \to 0 \right\}.$$

The ring $\mathbf{A}_K$ is a discrete valuation ring with uniformizer p, and residue field $\mathbf{A}_K/p = \mathbf{E}_K$. It is endowed with a Frobenius endomorphism

$$\varphi : \mathbf{A}_K \to \mathbf{A}_K, \qquad f(X) \mapsto f((1+X)^p - 1),$$

whose reduction mod p gives the usual Frobenius endomorphism of $\mathbf{E}_K$.

Definition 1.1. A φ-module is a finitely generated module M over $\mathbf{A}_K$ with a semilinear action of φ. We say that M is étale if the linearization of φ is an isomorphism.

For example, if M is free, and if we fix a basis with $A = \mathrm{Mat}(\varphi)$, then M is étale precisely when $p \nmid \det(A)$.

The main result concerning φ-modules is the following theorem of Fontaine.

Theorem 1.1. *There is an equivalence of categories*

$$\{\text{finitely generated } G_{\mathbf{E}_K}\text{-representations over } \mathbb{Z}_p\} \longleftrightarrow \{\text{étale } \varphi\text{-modules}\}.$$

Let us explain how this equivalence works. One constructs (as we shall do below) a large discrete valuation ring $\mathbf{A}$ with uniformizer p, such that $\mathbf{A}/p = \mathbf{E}_K$. The ring $\mathbf{A}$ is endowed with an action of $G_{\mathbf{E}_K}$ and with a Frobenius endomorphism φ. One has $\mathbf{A}^{G_{\mathbf{E}_K}} = \mathbf{A}_K$ and $\mathbf{A}^{\varphi=1} = \mathbb{Z}_p$. The functors

$$V \mapsto \mathrm{D}(V) = (\mathbf{A} \otimes_{\mathbb{Z}_p} V)^{G_{\mathbf{E}_K}}$$

and

$$\mathrm{V}(M) = (\mathbf{A} \otimes_{\mathbf{A}_K} M)^{\varphi=1} \leftarrow\!\shortmid M$$

then provide the asserted equivalence.

The details of the construction of $\mathbf{A}$ will not be used in the sequel, but we provide them here for the convenience of the reader. Let $\mathbb{C}_p$ be the completion of $\overline{\mathbb{Q}_p}$, and consider the ring $\varprojlim_{x^p \leftarrow x} \mathcal{O}_{\mathbb{C}_p}$, with operations

$$(x^{(n)})(y^{(n)}) = (x^{(n)}y^{(n)}) \quad \text{and} \quad (x^{(n)}) + (y^{(n)}) = (\lim_{k\to\infty}(x^{(n+k)} + y^{(n+k)})^{p^k}).$$

Then $\varprojlim_{x^p \leftarrow x} \mathcal{O}_{\mathbb{C}_p}$ is an integral domain, and its field of fractions, denoted by $\mathbb{C}_p^\flat$, is a complete, nonarchimedean and algebraically closed field of characteristic p. Let $\varepsilon = (\varepsilon^{(n)}) \in \mathbb{C}_p^\flat$ be any compatible sequence of p^n'th roots of unity, i.e. $\varepsilon^{(0)} = 1$. Fixing such a choice yields an embedding

$$\mathbf{E}_K = k((X)) \hookrightarrow \mathbb{C}_p^\flat, \quad X \mapsto \varepsilon - 1.$$

This embedding lifts to characteristic 0, so that for the Witt vectors ring $W(\mathbb{C}_p^\flat)$ we have

$$\mathbf{A}_K \hookrightarrow W(\mathbb{C}_p^\flat), \quad X \mapsto [\varepsilon] - 1.$$

One then defines $\mathbf{A}$ to be the henselization of $\mathbf{A}_K$ in $W(\mathbb{C}_p^\flat)$. The natural Frobenius endomorphism and $G_{\mathbf{E}_K}$-action on $\mathbb{C}_p^\flat$ ascend by functorially to $W(\mathbb{C}_p^\flat)$. These preserve $\mathbf{A}$ and thus endow it with the desired φ and $G_{\mathbf{E}_K}$-action.

2. Étale(φ,Γ)-Modules

In p-adic Hodge Theory one attaches certain semilinear invariants to subcategories of p-adic representations. In this section our goal will be to attach such an invariant to the category of all p-adic representations.

First we set up some notation. Let μ_{p^n} be the set of roots of unity of order p^n in $\overline{K}$. Let $K_n = K(\mu_{p^n})$ and $K_\infty = \bigcup_{n=1}^\infty K_n$. Finally, let G_K be the absolute Galois group of K and H_K the absolute Galois group of K_∞, and denote by $\Gamma_K = G_K/H_K$ the quotient. We have the cyclotomic character $\chi : \Gamma_K \to \mathbb{Z}_p^\times$ obtained through the composition

$$\Gamma_K = \mathrm{Gal}(K_\infty/K) \hookrightarrow \mathrm{Gal}(\mathbb{Q}_p(\mu_{p^\infty})/\mathbb{Q}_p) \xrightarrow{\sim} \mathbb{Z}_p^\times.$$

The ring $\mathbf{A}_K$ is endowed with an action of Γ_K, such that for $\gamma \in \Gamma_K$ we have the action

$$\gamma : \mathbf{A}_K \to \mathbf{A}_K, \quad f(X) \mapsto f((1+X)^{\chi(\gamma)} - 1).$$

This action commutes with the Frobenius endomorphism.

Definition 2.1. A (φ,Γ)-module is a finitely generated module M over $\mathbf{A}_K$ with a semilinear actions of φ and of Γ_K which commute. We say that M is étale if the linearization of φ is an isomorphism.

Remark. If M is free over $\mathbf{A}_K$, and if we fix a basis with $A = \mathrm{Mat}(\varphi)$ and $B = \mathrm{Mat}(\gamma)$ for $\gamma \in \Gamma_K$, the condition that φ and γ commute is equivalent to the identity $A\varphi(B) = B\gamma(A)$.

By a *p-adic representation* we mean a finitely generated G_K-representation over $\mathbb{Z}_p$ or over $\mathbb{Q}_p$. The main result needed to make the connection with p-adic representations is the following theorem, originally due to Fontaine and Winterberger.

Theorem 2.1. *There is a natural isomorphism* $H_K \xrightarrow{\sim} G_{\mathbf{E}_K}$.

Combining this with the Theorem 1.1 then yields:

Theorem 2.2. *There is an equivalence of categories*

$$\{p\text{-adic representations over } \mathbb{Z}_p\} \longleftrightarrow \{\text{étale } (\varphi,\Gamma)\text{-modules}\}.$$

Explicitly, the functors are given by

$$V \mapsto \mathrm{D}(V) = (\mathbf{A} \otimes_{\mathbb{Z}_p} V)^{H_F}$$

and

$$\mathrm{V}(M) = (\mathbf{A} \otimes_{\mathbf{A}_K} M)^{\varphi=1} \leftarrow\!\shortmid M.$$

This equivalence respects all the expected constructions on both sides, such as taking duals, tensor products, elementary divisors, etc.

It seems to be hard to explicitly compute the (φ,Γ)-module corresponding to a given p-adic representation. Here are a few examples where such a description is possible.

Example 2.1. Let $\delta : G_K \to \mathbb{Z}_p^\times$ be a character factoring through Γ_K (for example, one could take the cyclotomic character). Denote by $V(\delta)$ the rank 1 representation associated to δ. Then $\mathrm{D}(V(\delta)) = \mathbf{A}_K(\delta)$.

Example 2.2. ([Be2], discussion after Theorem 18.8) Suppose that $K = \mathbb{Q}_p$. By local class field theory, any character $G_{\mathbb{Q}_p} \to \mathbb{Z}_p^\times$ arises uniquely from a character $\delta : \mathbb{Q}_p^\times \to \mathbb{Z}_p^\times$. Let $\mathbf{A}_{\mathbb{Q}_p}(\delta)$ be the rank 1 (φ,Γ)-module with $\mathbf{A}_{\mathbb{Q}_p}(\delta) = \mathbf{A}_{\mathbb{Q}_p} e_\delta$, where

$$\varphi(e_\delta) = \delta(p) e_\delta \quad \text{and} \quad \gamma(e_\delta) = \delta(\chi(\gamma)) e_\delta \quad \text{for } \gamma \in \Gamma_{\mathbb{Q}_p}.$$

We will show that under the aforementioned equivalence of categories, $V(\delta)$ corresponds to $\mathbf{A}_{\mathbb{Q}_p}(\delta)$.

To see this, consider $\widehat{\mathbb{Z}_p^{\mathrm{un}}}$, the ring of integers of the maximal unramified extension of $\mathbb{Q}_p$. A successive approximation argument shows that the map

$$\widehat{\mathbb{Z}_p^{\mathrm{un}}}^\times \to \widehat{\mathbb{Z}_p^{\mathrm{un}}}^\times, \quad x \mapsto x/\varphi(x),$$

is surjective. In particular, there is an element $c \in \widehat{\mathbb{Z}_p^{\mathrm{un}}}^\times$ with $\varphi(c) = \delta(p^{-1})c$. It then follows that $\varphi(ce_\delta) = ce_\delta$, so $\mathbb{Z}_p \cdot ce_\delta = (\mathbf{A}(\delta))^{\varphi=1} = \mathrm{V}(\mathbf{A}_{\mathbb{Q}_p}(\delta))$.

A unit $u \in \mathbb{Z}_p^\times \subset \mathbb{Q}_p^\times$ acts on e_δ through $\delta(u)$ and trivially on c as it is unramified. Similarly, p^{-1} acts trivially on e_δ (the action on it factors through the cyclotomic character) and it acts on c by the arithmetic Frobenius, which is given by multiplying by $\delta(p^{-1})$ by construction. Thus $\mathbb{Z}_p \cdot ce_\delta = V(\delta)$.

Example 2.3. ([Be1]) This time we construct a two dimensional representation of $G_{\mathbb{Q}_p}$ over $\mathbb{Q}_p$ and describe the corresponding (φ,Γ)-module. Let $\mathbb{Q}_{p^2}$ be the unramified extension of $\mathbb{Q}_p$ of degree 2, and let $\mathbb{Q}_{p^2}^{\mathrm{LT}}$ be the Lubin-Tate extension of $\mathbb{Q}_{p^2}$ associated to the prime p, so that $\mathrm{Gal}(\mathbb{Q}_{p^2}^{\mathrm{LT}}/\mathbb{Q}_{p^2}) \cong \mathbb{Z}_{p^2}^\times$ through the Lubin Tate character. Then one has a natural isomorphism $\mathrm{Gal}(\mathbb{Q}_{p^2}^{\mathrm{LT}}/\mathbb{Q}_p) \cong \mathbb{Z}_{p^2}^\times \rtimes \mathbb{Z}/2\mathbb{Z}$, with the homomorphism $\mathbb{Z}/2\mathbb{Z} \to \mathrm{Aut}(\mathbb{Z}_{p^2}^\times)$ given by $\varepsilon \mapsto \sigma^\varepsilon$, σ being the arithmetic

Frobenius of $\mathbb{Z}_{p^2}$. Let $d \in \mathbb{Z}_p$ be an element such that $\mathbb{Z}_{p^2} = \mathbb{Z}_p[\sqrt{d}]$, then we have a homomorphism $\mathbb{Z}_{p^2}^\times \rtimes \mathbb{Z}/2\mathbb{Z} \to \mathrm{GL}_2(\mathbb{Z}_p)$ given by

$$(x + y\sqrt{d}, 0) \mapsto \begin{pmatrix} x & dy \\ y & x \end{pmatrix} \quad \text{and} \quad (x + y\sqrt{d}, 1) \mapsto \begin{pmatrix} x & -dy \\ y & -x \end{pmatrix}.$$

We then obtain a two dimensional representation V through the composition

$$\mathrm{Gal}(\overline{\mathbb{Q}_p}/\mathbb{Q}_p) \to \mathrm{Gal}(\mathbb{Q}_{p^2}^{\mathrm{LT}}/\mathbb{Q}_p) \cong \mathbb{Z}_{p^2}^\times \rtimes \mathbb{Z}/2\mathbb{Z} \to \mathrm{GL}_2(\mathbb{Z}_p).$$

A description of the corresponding (φ,Γ) is given as follows. Recall that Frobenius operator φ on $\mathbf{A}_{\mathbb{Q}_p}$ is defined by $\varphi(f(X)) = f((1+X)^p - 1)$. Let $Q_2 = \varphi(\varphi(X)/X)$. One may show that for $\gamma \in \Gamma$ there is a unique $f_\gamma(X) \in 1 + X\mathbb{Z}_p[\![X]\!]$ such that $\varphi^2(f_\gamma(X))/f_\gamma(X) = \gamma(Q_2)/Q_2$. Then there is a basis of $\mathrm{D}(V)$ for which

$$\mathrm{Mat}(\varphi) = \begin{pmatrix} 0 & 1 \\ Q_2 & 0 \end{pmatrix} \quad \text{and} \quad \mathrm{Mat}(\gamma) = \begin{pmatrix} \chi(\gamma)\varphi(f_\gamma(X)) & 0 \\ 0 & \chi(\gamma) f_\gamma(X) \end{pmatrix} \quad \text{for } \gamma \in \Gamma.$$

Here is an example of a (φ,Γ)-module for which the author is unaware of any computation of the corresponding p-adic representation.

Example 2.4. ([Sch, 2.3, Example C] and [Be-Li-Zhu]) Let $K = \mathbb{Q}_p$ and fix a two dimensional basis e_1, e_2 over $\mathbf{A}_{\mathbb{Q}_p}$. Define φ by

$$\mathrm{Mat}(\varphi) = \begin{pmatrix} 0 & -1 \\ \varphi(X)/X & 0 \end{pmatrix}.$$

To define the $\Gamma_{\mathbb{Q}_p}$-action, recall that one has the decomposition

$$\log(1+X) = X \prod_{n \geq 0} \frac{\varphi^{n+1}(X)}{p\varphi^n(X)}.$$

Indeed, both sides have exactly the same zeros, and up to a constant this completely determines a power series $\mathbb{Z}_p[\![X]\!]$, by the Weierstrass approximation theorem. Note that the $(\varphi, \Gamma_{\mathbb{Q}_p})$-structure extends to $\log(1+X)$. We have

$$\varphi(\log(1+X)) = p\log(1+X) \quad \text{and} \quad \gamma(\log(1+X)) = \chi(\gamma)\log(1+X).$$

Define

$$\log^+(1+X) = \prod_{n\geq 0} \frac{\varphi^{2n+2}(X)}{p\varphi^{2n+1}(X)} \quad \text{and} \quad \log^-(1+X) = \prod_{n \geq 0} \frac{\varphi^{2n+1}(X)}{p\varphi^{2n}(X)},$$

so that $\log(1+X) = X\log^+(1+X)\log^-(1+X)$. It is easy to check that $\varphi(\log^+) = \frac{pX}{\varphi(X)}\log^-$ and $\varphi(\log^-) = \log^+$.

Now for any $a \in \mathbb{Z}_p^\times$, the quotients $\log^+(1+X)/\log^+((1+X)^a)$ and $\log^-(1+X)/\log^-((1+X)^a)$ have no zeros, and therefore are units in $\mathbb{Z}_p[\![X]\!]$. Hence, we may define the $\Gamma_{\mathbb{Q}_p}$ action by

$$\gamma(e_1) = \frac{\log^+(1+X)}{\log^+((1+X)^{\chi(\gamma)})} e_1 \quad \text{and} \quad \gamma(e_2) = \frac{\log^-(1+X)}{\log^-((1+X)^{\chi(\gamma)})} e_2.$$

One may then check that the φ and $\Gamma_{\mathbb{Q}_p}$ actions commute, and that this gives rise to a (φ,Γ)-module.

3. Overconvergent (φ,Γ)-Modules

Let $\mathbf{A}_K^\dagger \subset \mathbf{A}_K$ be the subring of $\mathbf{A}_K$ defined by

$$\mathbf{A}_K^\dagger = \{f(X) = \sum_{n=-\infty}^{\infty} a_n X^n \in \mathbf{A}_K,\ f \text{ converges on some nonempty annulus } (r,1)\}.$$

The operations of φ and Γ descend to $\mathbf{A}_K^\dagger$, and we can define a notion of a (φ,Γ)-module over the ring $\mathbf{A}_K^\dagger$. This leads to the following definition.

Definition 3.1. An (φ,Γ)-module over $\mathbf{A}_K$ is *overconvergent* if it descends to a (φ,Γ)-module over $\mathbf{A}_K^\dagger$.

For example, for a free (φ,Γ) to be overconvergent it is neccessary and sufficient that there exists a basis and some r for which $\mathrm{Mat}(\varphi)$ has coefficients in which converge in some nonempty annulus $(r,1)$.

The main result concerning overconvergent (φ,Γ)-modules is the following theorem due to Cherbonnier-Colmez [Ch-Co].

Theorem 3.1. *Every étale (φ,Γ)-module is overconvergent. More precisely, base extension induces an equivalence of categories*

$$\{\textit{étale } (\varphi,\Gamma)\textit{-modules over } \mathbf{A}_K^\dagger\} \longleftrightarrow \{\textit{étale } (\varphi,\Gamma)\textit{-modules over } \mathbf{A}_K\}.$$

The *Robba ring* $\mathbf{B}_{\mathrm{rig},K}^\dagger$ is the ring given by

$$\mathbf{B}_{\mathrm{rig},K}^\dagger = \{f(X) = \sum_{n=-\infty}^{\infty} a_n X^n,\ a_n \in K,\ f \text{ converges on some nonempty annulus } (r,1)\}.$$

In other words, we have dropped the boundness condition in the definition of $\mathbf{A}_K^\dagger$. Note that $\mathbf{B}_{\mathrm{rig},K}^\dagger$ is endowed with a φ-action and a Γ-action, so that there is a notion of an étale (φ,Γ)-module over it.

As $\mathbf{B}_{\mathrm{rig},K}^\dagger$ is p-divisible, an étale (φ,Γ)-module over it could possibly encode any information about the torsion part of a p-adic representation. It turns out that this is the only limitation, however. Inverting p in all our rings, that is, replacing the rings $\mathbb{Z}_p$,$\mathbf{A}_K, \mathbf{A}$ by $\mathbb{Q}_p, \mathbf{B}_K = \mathbf{A}_K[1/p], \mathbf{B} = \mathbf{A}[1/p]$ respectively leads to categories of étale (φ,Γ)-modules in characteristic 0, and the analogues of the previous theorems still hold. In [Ke], Kedlaya proves the following result.

Theorem 3.2. *Base extension induces an equivalence of categories*

$$\{\textit{étale } (\varphi,\Gamma)\textit{-modules over } \mathbf{B}_K^\dagger\} \longleftrightarrow \{\textit{étale } (\varphi,\Gamma)\textit{-modules over } \mathbf{B}_{\mathrm{rig},K}^\dagger\}.$$

Combining this with Theorem 2.2 and the Cherbonnier-Colmez theorem, we have:

Corollary 3.1. *There is an equivalence of categories*

$$\{p\textit{-adic representations over } \mathbb{Q}_p\} \longleftrightarrow \{\textit{étale } (\varphi,\Gamma)\textit{-modules over } \mathbf{B}_{\mathrm{rig},K}^\dagger\}.$$

We denote the corresponding functor by $\mathrm{D}_{\mathrm{rig}}^\dagger$.

Remark. Both of $\mathbf{B}_K$ and $\mathbf{B}^\dagger_{\mathrm{rig},K}$ contain $\mathbf{B}^\dagger_K$, but neither of them is contained in the other. So it is really neccessary to consider overconvergent (φ,Γ)-modules if we want to relate p-adic representations with $\mathbf{B}^\dagger_{\mathrm{rig},K}$.

4. Applications

4.1. **p-adic Hodge Theory: Computing $\mathrm{D}_{\mathrm{cris}}(V)$ and $\mathrm{D}_{\mathrm{st}}(V)$ from $\mathrm{D}^\dagger_{\mathrm{rig},K}(V)$.** After extending scalars to $\mathbf{B}^\dagger_{\mathrm{rig},K}$ there is a very natural way to compute $\mathrm{D}_{\mathrm{cris}}$ and D_{st} directly from the (φ,Γ)-module. Let $t=\log(1+X)\in\mathbf{B}^\dagger_{\mathrm{rig},K}$; the point is that this element is a period for the cyclotmic character, i.e. $\gamma(t)=\chi(\gamma)(t)$ for $\gamma\in\Gamma_{\mathbb{Q}_p}$, just like a similar element lying in $\mathbf{B}_{\mathrm{cris}}$. Note that t does not lie in $\mathbf{A}_F$ or even in $\mathbf{A}_F[1/p]$, so it really is necessary to to work with the Robba ring to have this element.

Let $\mathbf{B}^\dagger_{\log,K}=\mathbf{B}^\dagger_{\mathrm{rig},K}[\log(X)]$ with the following actions on $\log(X)$:

$$\varphi(\log(X))=p\log(X)+\log(\varphi(X)/X^p)\ \text{ and }\ \gamma(\log(X))=\log(X)+\log(\gamma(X)/X)\ \text{ for } \gamma\in\Gamma_K.$$

We also define a monodromy map N on $\mathbf{B}^\dagger_{\mathrm{rig},K}[\log(X)]$ by $N=-p/(p-1)\cdot d/dlog(X)$. The following theorem is due to Berger ([Be3]).

Theorem 4.1. *Let V be a p-adic representation over $\mathbb{Q}_p$. Then* $\mathrm{D}_{\mathrm{cris}}(V)=\left(\mathrm{D}^\dagger_{\mathrm{rig}}(V)[1/t]\right)^{\Gamma_K}$ *and* $\mathrm{D}_{\mathrm{st}}(V)=\left(\mathrm{D}^\dagger_{\log}(V)[1/t]\right)^{\Gamma_K}$.

4.2. **Triangulline Representations.** For this section we will still be working with (φ,Γ)-modules over $\mathbf{B}^\dagger_{\mathrm{rig},K}$.

Although p-adic representations correspond only to those (φ,Γ)-modules which are étale, it is very useful to consider the larger category of all (φ,Γ)-modules. Even if a p-adic representation is irreducible, its corresponding étale (φ,Γ)-module may not be irreducible in the category of all (φ,Γ)-modules. This point of view is illustrated in the following definition.

Definition 4.1. Let V be a p-adic representation over $\mathbb{Q}_p$.

1. We say that V is *split triangulline* if $\mathbf{B}^\dagger_{\mathrm{rig},K}(V)$ is a successive extension of rank 1 (φ,Γ)-modules.

2. We say that V is *triangulline* if it is split triangulline after a finite extension of of its coefficients.

Twp important examples are the following.

Example 4.1. ([Co]) Semistable representations of $G_{\mathbb{Q}_p}$ are triangulline.

Example 4.2. ([Co, Ki]) Representations of $G_{\mathbb{Q}_p}$ which arise from overconvergent p-adic modular forms are triangulline.

Here is an explicit example of such a representation and its corresponding extension of rank 1 modules.

Example 4.3. ([Co, §4.5]) Consider the level 1, weight 12 modular eigenform $\Delta(z) = q\prod_{n=1}^{\infty}(1-q^n)^{24}$. Let y be a solution of the equation $y^2 - a_p y + p^{11} = 0$, and consider the characters $\lambda_1, \lambda_2 : \mathbb{Q}_p^\times \to \mathbb{Q}_p^\times$ defined by $\lambda_1(p^k u) = y^k$ and $\lambda_2(p^k u) = y^{-k}u^{11}$. Then we have the exact sequence

$$0 \to \mathbf{B}^{\dagger}_{\mathrm{rig},\mathbb{Q}_p}(\lambda_1) \to \mathrm{D}^{\dagger}_{\mathrm{rig}}(V_p(\Delta)^*) \to \mathbf{B}^{\dagger}_{\mathrm{rig},\mathbb{Q}_p}(\lambda_2) \to 0$$

describing $\mathrm{D}^{\dagger}_{\mathrm{rig}}(V_p(\Delta)^*)$ as an extension of two rank 1 (φ,Γ)-modules.[1]

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[1] The extension type is also known but we will not describe it here.