Problem 1. [A problem from Luis S.] Go to the website: http://www.math.uchicago.edu/~luis/pde/fd.html to compute the solutions to the following PDE up to time \( t = 0.1 \) and print out the resulting graph.

1. \( u_t = u_{xx}/20 \), with Dirichlet condition \( u(0,t) = u(1,t) = 0 \) for \( t > 0 \) and initial condition \( u(x,t) = x \sin(5\pi x) \).
2. \( u_t = u_{xx}/10 + 2u_x \), with Dirichlet condition \( u(0,t) = u(1,t) = 0 \) for \( t > 0 \) and initial condition \( u(x,t) = \sin(\pi x) \).
3. \( u_t = 2u_x \), with Neumann condition \( u_x(0,t) = u_x(1,t) = 0 \) for \( t > 0 \) and initial condition \( u(x,t) = (1-x) \sin(3\pi x) \). (Note that the Neumann condition on the left is essentially ignored. This equation is called the transport equation.)
4. \( u_t = -u_{xx}/250 \), with Dirichlet condition \( u(0,t) = u(1,t) = 0 \) for \( t > 0 \) and initial condition \( u(x,t) = \sin(\pi x)/5 \).
5. \( u_t = -u_{xx}/250 \), with Dirichlet condition \( u(0,t) = u(1,t) = 0 \) for \( t > 0 \) and initial condition \( u(x,t) = \sin(\pi x)/5 \) plus a tiny perturbation that you draw with the mouse anywhere (just one click).

Note: A quick intro to finite difference schemes for PDE. We are approximating the solution \( u(x,t) \) of a PDE on a finite interval, e.g. \([0,1]\), for times \( 0 \leq t \leq T \). We discretize space by a uniform grid of \( N \) points spaced by length \( h \) in \( x \) and \( M \) times spaced by width \( k \) in \( t \). We will define a finite difference approximation \( u[i,j] \) intended to approximate the value of \( u(ih,jk) \). You should discretize \( u_{xx}(ih,jk) \approx (u[i+1,j]+u[i-1,j]-2*u[i,j])/(h*h) \). The first derivative \( u_x \) can be discretized either as \( u_x(ih,jk) \approx (u[i+1,j]-u[i,j])/h \) or \( u_x(ih,jk) \approx (u[i+1,j]-u[i-1,j])/(2*h) \). Depending on the particular PDE one discretization may work better than another. A typical explicit scheme to approximate the solution of a heat equation would be,

\[ u[i,j+1] = u[i,j] + k \cdot (u[i+1,j]+u[i-1,j]-2\cdot u[i,j])/(h*h) \]

in this way, given the values of \( u[i,j] \) for \( 1 \leq i \leq N \), we could compute the values of \( u[i,j+1] \) and iterate. Note that in the scheme above we discretized \( u_t(ih,jk) \approx (u[i,j+1]-u[i,j])/k \). The word explicit means that the right hand side of the equation for \( u[i,j+1] \) depends only only the values of \( u[.,j] \) and not on \( u[.,j+1] \).