The test is cumulative, covering all the topics from the first and second exams and in addition the topics from Chapters 7.3-7.6 and 8.1.

- **General Advice**
  - Make your drawings very carefully, make them large enough to see the important details, clearly label the axes, clearly label several important points which help to interpret the scale.
  - Review what the domain and range of a function are if you are not clear on this. Understand the difference between a scalar valued function and a vector field. I am especially interested in checking that your basic understanding of the properties of the objects we are studying (scalars, vectors, and to a lesser degree matrices) is sound.

- **Chapter 1**
  - Vector operations (addition, subtraction, scalar multiplication) and their geometric interpretation.
  - The parametric equation of a line given (1) point and vector, (2) two points.
  - The inner product, the norm, their geometric interpretation.
  - Orthogonal projection.
  - Determinants of 2 × 2 and 3 × 3 matrices and their geometric interpretation (as area/volume of parallelogram and parallelopiped respectively).
  - Cross product, geometric interpretation, algebraic rules.
  - Triple product ((a × b) · c) relation with determinants of 3 × 3 matrices.
  - Equation of a plane in \( \mathbb{R}^3 \) given point and normal vector.
  - Distance from a point to a plane.
  - Definition of cylindrical and spherical coordinate systems and basic geometry, converting between coordinate systems.
  - Inner product in \( \mathbb{R}^n \), matrix multiplication and matrix vector multiplication.

- **Chapter 2**
  - The definition of a vector valued function, its domain and range.
  - The graph of a function, level sets, sections. Using the level sets and sections to produce an accurate drawing of the graph.
  - Open sets, definition of limits by neighborhoods.
  - Basic properties of limits.
  - Definition of continuity, basic properties, checking whether a function is continuous.
  - The partial derivative of a function of multiple variables.
  - Definition of differentiability for vector valued functions of several variables (i.e. \( f : \mathbb{R}^n \to \mathbb{R}^m \)), calculating the matrix \( Df \). What is a \( C^1 \) function on a domain \( U \subset \mathbb{R}^n \).
  - The tangent plane to the graph of a function \( f : \mathbb{R}^n \to \mathbb{R} \).
  - Relationship between differentiability and continuity.
  - Paths/curves in \( \mathbb{R}^n \) (what is the distinction?). Parametrizations, re-parametrizations of a curve by different paths.
  - Velocity vector of a path, speed of a path, tangent vector/line to a curve.
  - Differentiation rules, products, sums, constant multiples, quotients, and the chain rule.
  - Directional derivatives.
  - Understanding the gradient \( \nabla f \) as direction of fastest increase, normal to level surfaces of \( f \).
  - The equation of the tangent plane to a level surface of \( f : \mathbb{R}^n \to \mathbb{R} \).

- **Chapter 3**
  - What is a \( C^2 \) function, the mixed partial derivatives, equality of mixed partials.
  - Taylor’s Theorem up to second order for functions of several variables and its interpretation.
  - Local minima, maxima, critical points.
  - First derivative test for local extrema.
  - Quadratic functions in \( \mathbb{R}^n \) and the Hessian of \( f : \mathbb{R}^n \to \mathbb{R} \).
Positive/negative definiteness for a quadratic.

The second derivative test for local extrema, esp. the case \( n = 2 \) where it is much easier to check positive-definiteness.

Saddle points.

Strategy for finding the global maxima and minima of \( f : U \subseteq \mathbb{R}^n \rightarrow \mathbb{R} \) by parametrizing \( \partial U \).

The method of Lagrange multipliers for constrained extrema.

The method of Lagrange multipliers for finding the global maxima and minima of \( f : U \subseteq \mathbb{R}^n \rightarrow \mathbb{R} \).

Chapter 4

- Velocity, acceleration of a path.
- Differentiation rules for paths.
- Checking whether a path solves a differential equation like Newton’s Law \( ma(t) = mc''(t) = F(c(t)) \)
- Arc length of a path.
- Vector fields, gradient vector fields, flow lines of vector fields (drawing the vector field, drawing the flow lines, checking whether a path \( c \) is a flow line of a vector field \( V \)).
- Divergence of a vector field, curl of a vector field. How to compute them, what kind of functions are they (scalar, vector), basic facts like gradients have \( \nabla \times (\nabla f) = 0 \) for \( f \) scalar valued function on \( \mathbb{R}^3 \) and \( \nabla \cdot (\nabla \times F) = 0 \) for \( F : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) a vector field.

Chapter 5

- Cavalieri’s principle, reducing integral over a rectangle to iterated integrals.
- Fubini’s theorem (being able to apply it).
- Computing integrals over elementary regions in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \), changing the order of integration. You should be able to interpret in all directions between a description of the domain in words (e.g. the region of \( x^2 + y^2 + z^2 \leq 1 \) with \( z \geq 0 \)), an iterated integral over the domain, a description of the domain as an elementary domain in set theoretic notation.
- Tricks to compute integrals over non-elementary regions like splitting up into several elementary regions and summing.

Chapter 6

- Definitions of one-to-one, onto and invertible. Definition of the image of a set under a mapping.
- Images of linear maps (parallelograms are mapped to parallelograms), computed the area of a parallelogram (or parallelopiped) by the determinant of a matrix/linear map.
- The Jacobian determinant of a transformation.
- The change of variables theorem, computing integrals using linear changes of variables, polar, cylindrical and spherical coordinates.
- Expressing domains of \( \mathbb{R}^3 \) in different coordinate systems (closely related with computing integrals using change of variables).

Chapter 7

- The path integral, the line integral (what is the difference with path integral?). Computing these in various examples.
- Reparametrizations of paths and how the line/path integral can change under reparametrization.
- Line integrals of gradient vector fields.
- Line integrals over simple curves and over curves with several components.
- Parametrizations of surfaces, general concepts and specific common examples like parametrizing a plane, a graph of a function, a sphere, seeing when to use spherical or cylindrical coordinates to come up with parametrization.
- Tangent vectors \( T_u, T_v \) to a parametrized surface at a point, tangent plane to a parametrized surface at a point, regular points of a surface, normal vector and unit normal vector to a surface at a point.
- The area of a parametrized surface, the integral of scalar functions over surfaces.
- Oriented surfaces and surface integrals of vector fields on oriented surfaces.

Chapter 8
Green’s Theorem and its rephrasing as Stokes’ Theorem and the Divergence Theorem in $\mathbb{R}^2$. Applying this in a simple domain or in a domain with holes (making sure the line integrals on the boundary are oriented correctly, counter-clockwise for outer boundary clockwise for inner boundaries).