Indeed this property defines an irreducible hypersurface in $\mathrm{Sym}^{12}P$ invariant under $PGL_2\mathbb{C}$. Ravi Vakil computed its degree: 3762 and gave some other characterizations of it.

We will give this property some geometric content.
Let D be <u>any</u> reduced divisor of deg 12 on a proj. line P. Following Deligne-Mostow we consider the μ_6-cover of P totally ramified over D:

$$\begin{tikzcd} & C \arrow[d, "\mu_6"] \\ D \arrow[ur] \arrow[r, phantom, "\subset"] & P \end{tikzcd}$$

Galois cover total ram. over D.

(in affine coord: if D given by $f \in \mathbb{C}[z]$ of deg 12, then affine piece of C is given by $w^6 = f(z)$.) Deligne-Mostow assign to C with its μ_6-action the Jacobian $J(C)$ with its μ_6-action. They show that these data define a point of a 9-dim ball quotient $\Gamma_{DM}\backslash\mathbb{B}^9$ (an example of a Shimura variety), and that this point actually completely determines D up to proj. equivalence (the moduli space of 12 elt subsets of a proj line $(= \mathfrak{S}_{12}\backslash M_{0,12})$ is thus realized as a Zariski-open subset of $\Gamma_{DM}\backslash\mathbb{B}^9$.

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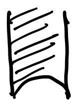
<u>Return to $PSL_2\mathbb{Z} \curvearrowright H_+$</u> Let $\Gamma := [PSL_2\mathbb{Z}, PSL_2\mathbb{Z}]$ has index 6 in $PSL_2\mathbb{Z}$ and $PSL_2\mathbb{Z}/\Gamma = (PSL_2\mathbb{Z})_{ab} \cong \mu_6$ (you can see that ~~from~~ the iso $PSL_2\mathbb{Z} = \mu_3 * \mu_2$). We get:

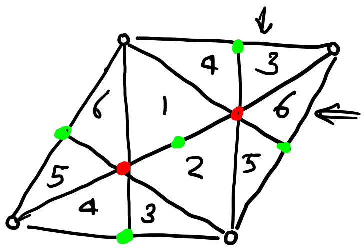
$$\begin{tikzcd} \Gamma\backslash\hat{H}_+ =: \hat{E} \arrow[d, "\mu_6"] \\ PSL_2\mathbb{Z}\backslash\hat{H}_+ = \mathbb{P}^1 \end{tikzcd}$$

It turns out that Γ is also transitive on $\mathbb{P}^1(\mathbb{Q})$, so the preimage of ∞ in E is just one point (a cusp of $\hat{E}$). One computes that E has genus 1. So it will be the ell curve with j-invariant 0:

$g(\hat{E}) = 1$ cusp = origin

The tesselation of $\hat{E}$ by the 6 copies of 

(here taking the shape of an equilateral triangle ) is as depicted (opposite sides identified)



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Back to the case of rational elliptic surface. Then $C_D \to P$ is the pull-back of $\hat{E} \to \mathbb{P}^1$.
We have a Cartesian diagram as below.

$$\begin{tikzcd} & C_D \arrow[r, "\tilde{J}"] \arrow[d, "\mu_6"] & \hat{E} \arrow[d, "\mu_6"] \\ D \arrow[ur] \arrow[r, phantom, "\subset"] & P \arrow[r, "J"] & \mathbb{P}^1 \end{tikzcd}$$

$D = J^*(\infty)$

Hence $\mathrm{Jac}(C_D)$ comes with a μ_6-projection onto $\mathrm{Jac}(\hat{E}) \cong \hat{E}$.
This property makes that under the Deligne-Mostow construction, $(\mathrm{Jac}(C_D), \mu_6)$ lands in an 8-dimensional ball quotient

$$\begin{tikzcd} \Gamma^\circ_{DM}\backslash\mathbb{B}^8 \arrow[r] & \Gamma_{DM}\backslash\mathbb{B}^9. \\ \mathbb{B}^8 \arrow[u] \arrow[r, phantom, "\subset"] & \mathbb{B}^9 \arrow[u] \\ \Gamma^\circ_{DM} \arrow[r] & \Gamma_{DM}. \end{tikzcd}$$

This gives a complex-hyperbolic structure on the moduli space of rational elliptic surfaces!