

Algebra 1 : Fifth homework — due Monday, November 17

Read through Chapter 7 of Fulton and Harris, and do the following exercises:

7.3, 7.5 (just state the answer; you don't need to give proofs), 7.7, 7.13, 7.14

(We discussed a lot of the examples in class; the main example that I didn't discuss, but that you will read about in this chapter, is the symplectic group.)

If you want to practice thinking about how topology and group theory interact, try exercise 8.1. (But you don't have to turn it in.)

Also do the following exercise:

1. Let G be a connected Lie group, and let \tilde{G} be its universal cover. Choose a point \tilde{e} lying over the identity e of G . Prove that there is a unique Lie group structure on \tilde{G} such that \tilde{e} is the identity, and such that the natural map $\tilde{G} \rightarrow G$ is a homomorphism. [Hint: I gave an intuitive argument for this in class, but you can make a formal argument in the following way. Use the universal property of the universal cover to lift the multiplication map $m : G \times G \rightarrow G$ to a (uniquely determined) map $\tilde{m} : \tilde{G} \times \tilde{G} \rightarrow \tilde{G}$ taking (\tilde{e}, \tilde{e}) to \tilde{e} . Now use uniqueness of lifts to show that the operation \tilde{m} is associative. Then play a similar game with inverses.]