Algebra 1 : Fifth homework — due Monday, November 7

Do the following exercises from Fulton and Harris:

7.2, 7.3, 7.4, 7.5, 7.7, 7.13, 7.14, 8.1

Also do the following exercise:

1. Let G be a connected Lie group, and let $\widetilde{G}$ be its universal cover. Choose a point $\tilde{e}$ lying over the identity e of G. Prove that there is a unique Lie group structure on $\widetilde{G}$ such that $\tilde{e}$ is the identity, and such that the natural map $\widetilde{G} \to G$ is a homomorphism. [Hint: Use the universal property of the universal cover to lift the multiplication map $m : G \times G \to G$ to a (uniquely determined) map $\tilde{m} : \widetilde{G} \times \widetilde{G} \to \widetilde{G}$ taking $(\tilde{e}, \tilde{e})$ to $\tilde{e}$. Now use uniqueness of lifts to show that the operation $\tilde{m}$ is associative. Then play a similar game with inverses.]