

p-adic schemes.

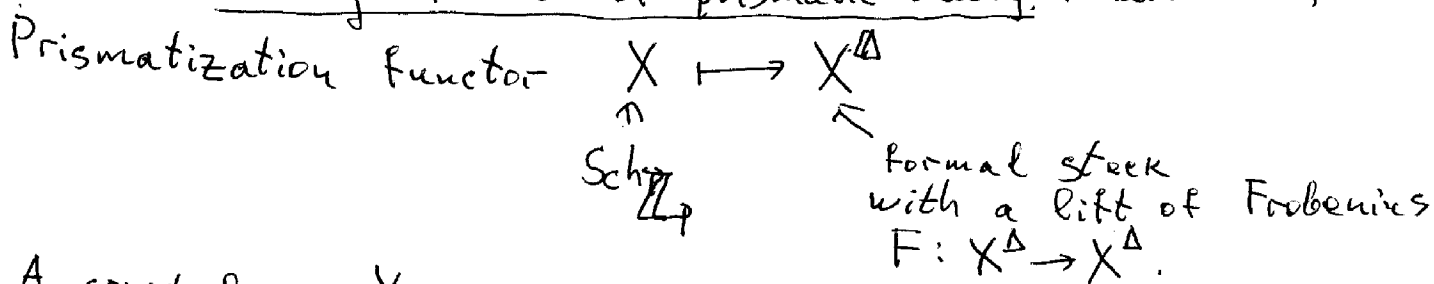
(-0-)

$$\text{Sch}_{\mathbb{Z}_p} = \{ \text{p-adic } \overset{\text{(formal)}}{\text{schemes}} \} = \varprojlim_n \text{Sch}/(\mathbb{Z}/p^n\mathbb{Z}).$$

A p-adic scheme is a compatible collection of  $(\mathbb{Z}/p^n\mathbb{Z})$ -schemes,  $n \in \mathbb{N}$ .  
One can consider a p-adic scheme as a ringed space (the underlying topol. space depends only on the reduction mod p).

Test rings (maybe postpone this?)  
 $\text{Rings}_p := \{ \text{Rings in which } p \text{ is nilpotent} \}$ . A p-adic scheme is a functor  $\text{Rings}_p \rightarrow \text{Sets}$  such that its restriction to  $\{ \mathbb{Z}/p^n\mathbb{Z}\text{-algebras} \}$  is representable by a scheme.

Stacky format of prismatic theory. (Bhatt-Lurie, not written).



A crystal on  $X$  is a complex of  $\mathcal{O}$ -modules on  $X^\Delta$ .  
An  $F$ -crystal is a pair  $(M, F^*M \rightarrow M)$  with a certain property: roughly,  $\text{Cone}(F^*M \rightarrow M)$  is supported on the "Hodge-Tate" locus of  $X^\Delta$ .  $F$ -crystals are p-adic analogs of  $\mathbb{Z}_p$ -sheaves.

$$f: X \rightarrow Y \text{ gives } f^\Delta: X^\Delta \rightarrow Y^\Delta.$$

$$f_*^\Delta: \{ F\text{-crystals on } X \} \rightarrow \{ F\text{-crystals on } Y \}. \text{ This is prismatic cohomology.}$$

Last quarter we discussed  $X^\Delta$  for  $X$  over  $\mathbb{F}_p$ . Then we get crystalline cohomology. Recall that  $(\text{Spec } \mathbb{F}_p)^\Delta = \text{Spf } \mathbb{Z}_p$ .

Before discussing the general case, ~~let me~~ define  $\Sigma = (\text{Spf } \mathbb{Z}_p)^\Delta$ . This is a stack, not a (formal) scheme.