1. Show that the set of sequence numbers is representable.

2. Let $R$ be a representable unary relation and let $g$ and $h$ be representable unary functions. Show that

$$f(a) = \begin{cases} g(a) & \text{if } a \in R \\ h(a) & \text{if } a \notin R \end{cases}$$

is representable.

3. Let $T$ be a decidable consistent theory (in an effective language). Show that there is a complete decidable consistent theory $C \supseteq T$. [Hint: List the sentences $\sigma_0, \sigma_1, \ldots$ in the language and for each sentence in turn, add it or its negation to $C$, in such a way as to maintain consistency and decidability.]