Math 27800 / CS 27800, Winter 2021: Assignment 5
Due Friday, February 19th

Problem 4 is a “take-home exam” problem, to be worked out individually.

1. Show that a theory is computably enumerable iff it is axiomatizable. [Hint: Any formula $\varphi$ is equivalent to $\varphi \land \varphi \land \cdots \land \varphi$.]

2. Show that the theory of the natural numbers has $2^{\aleph_0}$ many nonisomorphic countable models. [Hint: For each set of primes $A$, construct a model containing an element whose prime divisors are exactly the elements of $A$.]

3. Show that if $T$ is a consistent, $\omega$-complete theory (as defined in the previous assignment) in the language of $\mathfrak{N} = (\mathbb{N}; 0, S, <, +, \cdot, E)$ and $A_E \subseteq T$, then $T = \text{Th } \mathfrak{N}$.

4 (take-home exam problem). For $X \subseteq \mathbb{N}$, let

$$S_X = \{x : 2x \in X \text{ or } 2x + 1 \in X\}.$$ 

a. Show that if $X$ is representable then so is $S_X$.

b. Give an example of an $X$ such that $S_X$ is representable but $X$ is not. [Hint: Recall that every representable relation is computable.]