1. Show that the $<$ relation is not definable in $\mathcal{N}_S = (\mathbb{N}; 0, S)$. (Use the fact that the theory of $\mathcal{N}_S$ admits elimination of quantifiers.)

2. Show that if $T$ is a consistent, $\omega$-complete theory (as defined in the previous assignment) in the language of $\mathcal{N}$ and $A_E \subseteq T$, then $T = \text{Th} \mathcal{N}$. [Here $\mathcal{N} = (\mathbb{N}; 0, S, <, +, \cdot, E)$].

3. Show that the theory of the natural numbers has $2^{\aleph_0}$ many nonisomorphic countable models. [Hint: For each set of primes $A$, construct a model containing an element whose prime divisors are exactly the elements of $A$.]