1. Find the derivative of each of the following functions.

(a) \( f(x) = 3x^2 - 7 + \frac{4}{x} \) \( f'(x) = 6x - \frac{4}{x^2} \)

(b) \( f(x) = \sqrt{xe^x} \) \( f'(x) = \sqrt{xe^x} + \frac{e^x}{2\sqrt{x}} \)

(c) \( f(x) = \frac{\ln x - x}{x^2} \) \( f'(x) = \frac{x + 1 - 2\ln x}{x^3} \)

(d) \( f(x) = \ln(3x^3) \) \( f'(x) = \frac{3}{x} \)

(e) \( f(x) = x\sqrt{1-x^2} \) \( f'(x) = -\frac{x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} \)

(f) \( f(x) = e^{x/(x^2+1)} \) \( f'(x) = \frac{1-x^2}{(x^2+1)^2}e^{x/(x^2+1)} \)

(g) \( f(x) = \sqrt{e^{x^2} + 1} \) \( f'(x) = \frac{xe^{x^2}}{\sqrt{e^{x^2} + 1}} \)

(h) \( f(x) = \frac{(4-x)^8}{(x+1)(x-2)} \) \( f'(x) = \frac{-8(4-x)^7(x+1)(x-2) - (4-x)^8(2x-1)}{(x+1)^2(x-2)^2} \)

2. Explain why each of these functions is not differentiable at \( x = 0 \).

(a) \( f(x) = \ln x \)

\( f \) is not continuous at 0 since \( \ln 0 \) is undefined, so it is not differentiable at 0.

(b) The absolute value function \( f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \)
The absolute value function has a sharp point at \( x = 0 \). To see this algebraically, consider the limit definition of the derivative at \( x = 0 \). Approaching from the left, 
\[
\lim_{h \to 0^-} \frac{|0 + h| - |0|}{h} = -1.
\]
Approaching from the right, 
\[
\lim_{h \to 0^+} \frac{|0 + h| - |0|}{h} = 1.
\]
We can see that the derivative \( f'(0) = \lim_{h \to 0} \frac{|0 + h| - |0|}{h} \) does not exist.

(c) \( f(x) = x^{1/5} \)

The power rule formula yields \( f'(x) = \frac{1}{5x^{4/5}} \), which is undefined at \( x = 0 \) but approaches infinity as \( x \) approaches 0. Thus \( f \) has a vertical tangent line at \( x = 0 \), so it is not differentiable there.

3. The length of the side of a cube is measured to be 10 cm, with an error of \( \pm 0.1 \) cm (that is, the true length could be anywhere between 9.9 cm and 10.1 cm). Use linear approximation to estimate the minimum and maximum volume of the cube. You can compare your approximations to the actual minimum volume of \( 9.9^3 = 970.299 \text{ cm}^3 \) and maximum volume of \( 10.1^3 = 1030.301 \text{ cm}^3 \).

We will use two different linear approximations to the volume of a cube function \( f(x) = x^3 \) at the point \( x = 10 \). Taking \( \Delta x = 0.1 \) for the approximation gives us a maximum volume of 1030 \text{ cm}^3, and using \( \Delta x = -0.1 \) results in a minimum volume of 970 \text{ cm}^3. Such an estimate for the volume of the cube would be commonly written as 1000 \( \pm 30 \text{ cm}^3 \).